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THE GHOST IN THE MACHINE: HILBERT'S FORMALIST DREAM AND THE GÖDELLIAN INCOMPLETENESS OF HUMAN SOCIETY

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Abstract
This paper explores how we can move from David Hilbert's formalist way of thinking to Kurt Gödel's alternative approach by applying it in human social systems. Hilbert's hope to put the entirety of mathematics on a complete and consistent foundation by means of a formal systematization is officially as far as Enlightenment rationalism could possibly go. However, in 1931 Gödel presented his incompleteness theorem which proves that every formal system contains truths about mathematics that the system will never be able to prove from the inside out. Thus the incompleteness theorem of Godel effectively brings down the dream of a closed logical universe or formalism that validates itself. This paper suggests that the "Gödellian" limit is not just a technical limitation of mathematical logic, but an ontological property characterizing human society. By examining the shortfalls of formalism in communication, law, and governance we show that human systems are "open" by design, forever dependent on interpretation or judgment beyond the written rules. The "Gödellian sentence" of society is human intuition and context and creativity, the irreducible part of what makes us intelligent cannot be captured in formal systems.

Keywords: Hilbert, Godel, Formalism, incompleteness theorem, human society,

1. Introduction

The early twentieth century was a time of immense intellectual ferment, marked by a profound "crisis of foundations" in mathematics and logic. One of the most profound foundations was the discovery of Russell's paradox which had shaken the very bedrock of mathematical certainty.[1] In response to this instability, the mathematical community sought a methodology that could guarantee absolute objectivity and banish contradiction forever. The most prominent and ambitious response was the formalist program spearheaded by the eminent German mathematician David Hilbert. His goal was to strip mathematics of its reliance on metaphysical "intuition" and rebuild it as a rigorous, self-contained game of symbol manipulation governed by a finite and complete set of axioms. The formalist philosophy holds that mathematical statements are not about any particular subject matter but are rather the logical consequences of certain axioms and rules of inference. This drive toward formalism was not an isolated academic pursuit; it mirrored a broader societal and modernist impulse toward mechanization, bureaucracy, and the belief that all human affairs could, in principle, be optimized through closed, rule-based systems.

However, this vision of totalizing formal clarity was shattered by Godel's 1931 publication on incompleteness theorem. Incompleteness theorem shows that truth and provability are not synonymous such that in every formal system, there are propositions which the formal system cannot prove.[2] This paper explores the "Gödellian" fracture in Hilbert's program and its profound, though often underappreciated, implications for human society. We contend that the attempt to apply strict formalism to the context-rich reality of human interaction whether in communication, legal interpretation, or digital governance inevitably encounters a similar limit, a boundary that only human agency and interpretation can transcend.

2. Hilbert's Formalist Program

Formalism was the driving force and vision of David Hilbert. This project aligns with the most audacious project in the history of logic which is anchored on the foundation of mathematics. Building on his 1899 work, *Grundlagen der Geometrie* (Foundations of Geometry), where he demonstrated how traditional geometric intuition could be replaced by a purely formal axiomatic structure, Hilbert sought to do the same for all of mathematics.[3] His subsequent "Hilbert's

[1]Bertrand Russell discovered the paradox in 1901. It can be illustrated by considering a set containing all sets that do not contain themselves. Does this set contain itself? If it does, it violates its own definition. If it doesn't, it should be included. This contradiction revealed deep problems in the foundations of logic and set theory.

[2] Kurt Gödel, "Über formal unentscheidbare Sätze der Principia Mathematica und verwandter Systeme I," *Monatshefte für Mathematik und Physik* 38, no. 1 (1931): 173–98.

[3] David Hilbert, *Grundlagen der Geometrie* (Leipzig: Teubner, 1899).

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Program" aimed to formalize every branch of mathematics and, crucially, to provide a "finitist" proof of its consistency, a proof that used only simple, concrete, and unobjectionable reasoning about finite collections of symbols.[4]
The formalist program was built upon three primary pillars:

1 **Consistency**: A proof that no contradiction (e.g., proving both a statement and its negation) could ever be derived from the axioms.
2 **Completeness**: The assurance that every well-formed statement within the system could be either proven true or proven false.
3 **Decidability**: as much as completeness and consistency were of high interest to formalism, decidability was interested in creating effective algorithm or a procedure that is powerful enough to determine either truth value or falsehood of mathematical statements.

As the philosopher of mathematics Richard Zach has observed, Hilbert thought that if one showed that mathematics could be reduced to a soulless system of arbitrary symbols manipulated according to determinative rules, then its very foundation would not be reliant on human subjectivity and faulty intuition". This was a "rigorous formalism" that conceived of the mathematical universe as a finite, self-contained machine. If Hilbert had been right, there would have been no "unsolvable" problems in mathematics — every question has an answer and that answer can be found by churning through the rules. There would have been no place for the "ghostly" existence of truths not subject to proof.

3. The Structural Fracture: Gödel's Incompleteness Theorem

Hilbert made great progress in his dream about formalism which anchored on completeness and self-validating mathematical structure. Unfortunately, this dream was greatly punctured by Kurt Godel in his 1931 work, "On Formally Undecidable Propositions of Principia Mathematica and Related Systems I". Through a brilliant and complex feat of logic, Gödel introduced two theorems that fundamentally and permanently altered our understanding of formal systems.[5]

Table 1: Summary of Gödel's Incompleteness Theorems

Theorem	Description	Implication for Formalism
First Theorem	In any consistent formal system powerful enough to express arithmetic, there are statements that are true but unprovable within the system.	Formal systems are inherently "incomplete"; truth transcends the reach of any single set of axioms.
Second Theorem	A consistent formal system cannot prove its own consistency using only its own internal rules.	A system cannot "validate" itself from within; proving consistency always requires stepping outside to a broader, more powerful "meta-system."

Gödel's work demonstrated that "truth" is a richer, more expansive concept than "provability."[6] The fracture he identified was not a flaw to be patched but a fundamental structural property of all formal systems of sufficient complexity. The more powerful a system becomes, the more "Gödellian sentences" it generates statements that, through a line of reasoning external to the system, we can recognize as true, but which the system itself is blind to. This realization marked the end of the search for a single, all-encompassing "theory of everything" in mathematics. It suggested that human reasoning possesses a non-algorithmic quality, an ability to ascend to a meta-level and perceive truths that escape the rigid confines of any given mechanical logic.

Beyond the Blackboard: Formalism falls short in human society

Formalism in Hilbert and the incompleteness theorem in Godel are two extremes in modern logic. They create a form of conflict minding the constructions of mathematical systems in Hilbert and Godel's proof of limitations aren't simply a matter for the rarefied realm of numbers but reflects the dynamism of human society. It turns up daily in the "human

4 Raatikainen, Panu. "Gödel's Incompleteness Theorems." In *The Stanford Encyclopedia of Philosophy*, edited by Edward N. Zalta. Fall 2013. https://plato.stanford.edu/entries/goedel-incompleteness/.
5 Raatikainen, Panu. "Gödel's Incompleteness Theorems." In *The Stanford Encyclopedia of Philosophy*, edited by Edward N. Zalta. Fall 2013. https://plato.stanford.edu/entries/goedel-incompleteness/.

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machine" of society, where the well-meaning effort to make things more formally efficient and transparent and predictable almost invariably collides with the irreducible complexity of life as experienced.
Godel's work shows a non acceptance of strict formalism in mathematics where truth becomes synonymous with provability[7]. Godel demonstrated the inadequacy of formalism, hence argued that there are obviously some truths of mathematics that are not exclusively deduced from formal approach[8].
In the same vein, the metaphysics of Aquinas shows that the human person is a sum total of his experience, culture, intellect, environment, etc. More so the intellectual activity of human beings has both strict logical argumentation and intuition or the *theoria*[9]. Here strict logical argumentation implies formalism, syllogism, deductive or inductive reasoning while intuitive or *theoria* [10] allows insights, creativity, imaginations, etc. The dynamics of human experiences, communication, legal process, culture, intellect, environment, or intuition go a long way to expose the inadequacies of formalism and mathematical claims.

4.1 Communication and Meaningfulness Ambiguity
At the middle of the 20th century, the Shannon-Weaver communication model sought to reduce human interaction to a straight line process of encoding, transmission and decoding of a signal. Shannon-Weaver model of communication was developed in 1948 as a linear, technical model designed to explain how information flows from a source to a destination. This formal system of communication has about eight components such as; source, encoder, channel, noise, decoder, receiver, destination, and feedback[11].
This is a formal system where the source produces a message constructed in the form of a signal sent through a channel. In the same vein, the receiver in turn translates the message and makes it available to the destination. An example of this situation is the Shannon-Weaver model of communication. This model makes use of cell phone as transmitter and produces electric signals which are transmitted through wires, forming channels/electromagnetic waves/radiation to the destination phone of the receiver[12].
As revolutionary as this was in the fields of engineering and information theory, it was woefully inadequate for representing the subtlety of human meaning, hence it cannot comprehensively represent information from the source to the receiver. This challenge of inadequacy was already acknowledged by Shannon-Weaver's three types of problems of communication experienced herein; technical, semantic, and effectiveness problems. Here, technical problem is concern with how to use a signal to reproduce a message accurately from one location to another minding distortions such a noise which might reconstruct the source's original intentions[13].
As the communication theorist John Inah Ukam argues, human communication is an "open system" characterized by contextual sensitivity, non-verbal cues, emergent creativity, and shared, often unspoken, understanding.[14] A purely formal model of communication is "Gödellian" in its incompleteness: it can describe the syntax of a message but remains blind to the subtext, irony, and implicature that constitute the heart of human connection. The meaning of "I'm fine" cannot be formally deduced without access to a rich, external context of tone, relationship, and situation. Thus it is obvious here that the ambiguity between communication and meaningfulness is beyond the scope of formal systems of arithmetic models.

4.2 Legal Formalism and the Limits of Governance
In the realm of law, the philosophy of "strict formalism" seeks to reduce legal interpretation to the "plain meaning" of the text, treating law as a closed, axiomatic system. However, as legal scholar Charles E. A. Lincoln IV demonstrates, legal systems inevitably encounter "Gödellian" limits when faced with novel human situations that the original "axioms" (the laws as written) did not anticipate.[15] The recent advent of Decentralized Autonomous Organizations (DAOs), governed by the principle of "code is law," further highlights this inadequacy. As Andrea Cesaretti notes, while formal smart contracts provide unparalleled efficiency, they lack the "adaptive governance" required to handle unforeseen ethical

[7] Gödel, K. (1931). *On Formally Undecidable Propositions of Principia Mathematica and Related Systems I.* (London: Dove Publications), 12.
[8] Gödel, K. (1931). *On Formally Undecidable Propositions of Principia Mathematica and Related Systems I,* 174
[9] Driessen, A. (1989). *Philosophical Consequences of the Godel Theorem,* (Netherland: University Press), 23.

[10] Driessen, A. (1989). *Philosophical Consequences of the Godel Theorem,* 23.

[11] Shannon, C. E. (1948). "A Mathematical Theory of Communication". *Bell System Technical Journal.* 27(3): 381.
[12] Shannon, C. E. (1948). "A Mathematical Theory of Communication". *Bell System Technical Journal.* 27(3): 381.
[13] Narula, U. (2006). *Handbook of Communication Models, Perspectives, Strategies.* (Atlantic: University Press). 26.

14 John Inah Ukam, "Hilbert & Goedel on Completeness and Incompleteness Theory: A Look at the Problem of Formalism in Communication," *CRUTECH Journal of Communication* 4, no. 3 (2023), https://unicross.edu.ng/research/file/cd463e8fbd9ec5c88f91ab4596dbad301753042902.pdf.
[15]. Charles E. A. Lincoln IV, "Axiomatic Shifting Paradigms: Wittgenstein's Language-Games, Gödel's Incompleteness Theorem, Language, Law, and the Limits of Formalism," *University of Arkansas at Little Rock Law Review,* 47, no. 2 (2025), https://lawrepository.ualr.edu/lawreview/vol47/iss2/1/.

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dilemmas, systemic shocks, or the need for compassionate exceptions.[16] A rigid, purely formal legal system is trapped in a Gödellian bind: it is either "consistent" (but unable to render justice in novel cases) or it becomes "complete" only by introducing the very human discretion and interpretation it was designed to eliminate.

5. Synthesis: The Human Element as the "Gödellian Sentence"

The repeated inability of formalism within human society points to the "human element" itself as providing the final Gödellian sentence. This is analogous to how a logical system cannot demonstrate its own consistency but must rely on assumptions external to it, so can a social order not complete its own sanction and perpetuation through formal rules alone. It needs to rely on a constant appeal to an outside frame, the frame of human intuition, of moral judgment and of the ability to step "outside the system" and assess its ends and its consequences. Such is the mystery of wisdom.

Seen in this way, incompleteness is not a defect but an essential defense for human freedom and dignity. Participant action and outcome are pre-determined by a formal calculus in this "actual" social system, which is nothing other than totalizing to the point of being not human at all. By accepting the Gödellian fact of all our institutions, we recognize that meaning is not something "computed", but is and always remains something "constructed" in and through a process which is never completed nor closeable by any criterion, but only hammered open again and again through human interaction and judgment.

6. Conclusion

While Hilbert's formalism defends the rubrics of arithmetic laws, Gödel's incompleteness theorem shows lost of inadequacies in formalism claims. Thus incompleteness theorem punctures the rigidity of formalism beyond the blackboard. This is manifest in daily experience minding efforts to make things more formally efficient and transparent and predictable almost invariably collides with the irreducible complexity of life as experienced/lived. Godel rejects formalism in mathematics where truth becomes synonymous to provability[17] as this does not cohere with human society/experience. Formalism in Hilbert is oblivious of wholeness of human experience as expressed in the metaphysics of Aquinas where the human person[18] is a sum total of his experience, culture, environment, or intuition , etc.

Our intellectual development from Hilbert's confidence to Gödel's realism is a profound arrival of the human intellect. We have learned that unattainable finality is a beautiful goal, but only a goal. In the complex, adaptive systems of human society, the insufficiency of formalism is not a bug to be corrected but a feature to be appreciated. Whether we are designing AI, writing laws or constructing digital communities, tight formalism is something to struggle against. Rather, we must strive to create systems that allow room for the Gödellian "ghost" the unformalizable, creative and context-bound spirit of humankind as it is and must be the ultimate judge of truth, justice and meaning.

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16. Andrea Cesaretti, "Beyond the Code: Gödel's Incompleteness and the Limits of Formal Governance in DAOs," in *DAO Governance in Theory and Practice: Metrics and Frameworks* (Springer, 2025), https://link.springer.com/chapter/10.1007/978-3-032-09675-3_6.

17 Gödel, K. (1931). *On Formally Undecidable Propositions of Principia Mathematica and Related Systems I.* (London: Dove Publications), 12.

18 Driessen, A. (1989). *Philosophical Consequences of the Godel Theorem,* (Netherland: University Press), 23.