CHAPTER 16

PEDAGOGICAL INNOVATIONS IN SECONDARY SCHOOL MATHEMATICS

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Introduction

Pedagogy is referred to as the art, science, and practice of teaching. It comprises the methods, strategies, and principles used by teachers to facilitate learning and promote student growth. Teachers are facilitators who enable students' development and learning. The teacher uses different kinds of methods to activate every student in the classroom. It is therefore part of the teacher's professional responsibility to choose the best possible pedagogical solution for different groups and situations. Innovation is the process of introducing new practices, making changes, and as well bringing about new ideas. Pedagogy innovations in secondary school mathematics involve the process of introducing new teaching strategies/approaches and methods into the mathematics classrooms in secondary schools.

The teaching of mathematics today has become increasingly challenging. This is because Secondary school learners (students) in Nigeria are more passive than creative thinkers. Students' ability to think critically and fast as well as act is very crucial in learning of mathematics (Fennema, & Franke, 1992). In line with this concern, there is an urgent need for the pre-service teacher to be vast in knowledge of the content of mathematics subject matter. There is need to be creative in employing novel and innovative pedagogy that will motivate first of all the student teacher and students on the learning of Science Technology Engineering and Mathematics (STEM) subjects. The 21st-Century teacher needs to be more conversant with new instructional approaches that are basically student-centred and task-based. Pre-service teachers should organize learners into small groups, working as a whole class or even working individually to incorporate new ways of learning. That is, pre-service teachers need to be equipped with the knowledge and belief about mathematics as well as knowledge on how to teach mathematics in a way that would create a positive attitude that to make them see or perceive the usefulness of mathematics in their future life (Gómez, Bacelo, Marbán & Palacios, 2023). As a teacher, using new approaches that will make the teaching and learning of mathematics more concrete and practical is what the pre service teacher needs in order to teach well and makes his teaching interesting, enjoyable and exciting to students. It is believed that this book will be useful to the student teachers undergoing teacher training in our Universities and Colleges of Education in Nigeria.

Mathematics education has changed a lot over the years but many traditional methods are still the same. Historically, mathematics teaching in secondary schools has been; (a) lecture based – the teacher is the source of all knowledge and students are passive recipients of information. This method focuses on rote memorisation and procedural knowledge rather than conceptual understanding, (b) procedural fluency – students learn to follow steps to solve problems. While this is important it often comes at the expense of deeper understanding of mathematical concepts and their real life applications, (c) limited use of technology – traditional tools like textbooks, blackboards and simple calculators dominated the educational landscape with little integration of more advanced digital resources, (d) teacher centered classroom – the classroom is teacher centered where the teacher directs all activities and students have limited opportunity to collaborate or explore independently.

The evolving educational landscape and the demands of the 21st century necessitate a shift from these traditional methods to more innovative pedagogical approaches. The historical context of mathematics education highlights the persistence of traditional methods that are increasingly inadequate in meeting the needs of today's students. The demand for innovation is driven by the necessity to engage a diverse student population, develop essential skills, integrate technology, foster a growth mindset, encourage collaboration, adapt to changes, and reignite student interest in mathematics.

Innovations in pedagogy have become extremely essential in improving the quality of secondary school mathematics education. This chapter begins with an introduction that sets the stage for understanding the significance of pedagogical innovations in secondary school mathematics. It outlines the primary goals of the chapter, highlighting the need for changes in traditional teaching methods to better meet the needs of contemporary learners. It also delves into recent advancements in teaching secondary school mathematics and how they influence student engagement, understanding, and achievement. The key innovations discussed include the integration of technology, such as interactive software and online resources, which provide dynamic and personalized learning experiences. Collaborative learning approaches, like group problem-solving and peer teaching, have been proven to foster a deeper understanding and critical thinking skills. Additionally, inquiry-based learning methods encourage students to explore mathematical concepts through real-world applications, promoting a more meaningful and contextual grasp of the subject. The use of formative assessments and feedback has also evolved, enabling more tailored and responsive teaching strategies. These pedagogical innovations collectively contribute to a more engaging, effective, and inclusive mathematics education in secondary schools, preparing students for advanced studies and real-life problem-solving. By integrating technology, fostering collaboration, encouraging inquiry, and providing personalized feedback in secondary school mathematics, teachers can create a more engaging, effective, and inclusive learning environment.

Objectives
By the end of this chapter, pre-service teachers should be able to:

- Explain why pedagogical innovation is critical in Secondary School mathematics
- Demonstrate technology based strategies for teaching mathematical topics
- Design a collaborative activity for a lesson in any mathematical class.
- Apply formative assessment techniques to provide real-time feedback during classes.

The need for Pedagogical Innovations in Secondary School Mathematics

1. **Developing Critical Thinking and Problem-Solving Skills:** The modern world requires individuals who can think critically, solve complex problems, and adapt to new situations. Innovative pedagogies that promote higher-order thinking skills are essential to prepare students for the challenges of the future.

   - **E.g.:** While teaching quadratic equation to SS2 students

   "Let $f(x) = ax^2 + bx + c$ with $a \neq 0$. Prove that f(x) has two distinct real roots if and only if its discriminant $b^2 – 4ac > 0$. Then, given $f(x) = 2x^2 -3x +1$, determine whether it has two real roots and justify your answer".

By asking students to both prove a general theorem and then apply it to a concrete quadratic, you engage them in higher-order reasoning and adaptive transfer of knowledge to a new instance.

2. **Engaging a Diverse Student Population:** Today's classrooms are more diverse than ever, with students from various cultural, linguistic, and socio-economic backgrounds. Innovative teaching methods are needed to address the diverse learning styles and needs of all students, ensuring equity and inclusivity in mathematics education.

- **E.g.** While helping SS3 students prepare for WAEC

"Provide three different representations (algebraic, graphical, and verbal) of the exponential model $y = 2\,.\,3^x$. In small groups, have each student explain one representation and teach it to their peers, then reflect on which approach helped them understand the model best."

Offering multiple entry-points (symbolic, visual, and narrative) honors varied learning preferences; peer-teaching ensures that students both learn and articulate in their strongest mode, fostering inclusivity.

3. **Incorporating Technology and Digital Literacy:** With the rapid advancement of technology, integrating digital tools into mathematics education is crucial. Technology can enhance learning experiences, provide interactive and personalized instruction, and prepare students for a technologically driven world.

- **E.g.** In teaching graphing of quadratic equation to SS2 students,

"Use GeoGebra to plot y= $x^2 - 4x + 3$. Then dynamically adjust the coefficients a, b, c with sliders to observe how the parabola's vertex and roots change. Record three sets of (a, b, c) values and their corresponding roots."

Interactive sliders let students experiment in real time, build intuition about parameter effects, and deepen conceptual understanding while simultaneously developing their digital-tool fluency.

4. **Fostering a Growth Mindset:** Traditional methods often contribute to a fixed mindset, where students believe their abilities are static. Innovative teaching strategies can foster a growth mindset, encouraging students to embrace challenges, learn from mistakes, and develop a lifelong love for learning.

- **E.g.** While teaching SS3 students physical implementation of logarithms

"Assign an iterative approximation of √2 using Newton's method

: $x_{n+1} = \frac{1}{2}(x_n + \frac{2}{x_n})$. Have students log each iteration, note when they make errors and write a short reflection on what they learned from correcting their mistakes."

By framing "errors" as data points in an iterative process and requiring reflection on corrections, students learn that mistakes drive deeper learning – a core growth mindset principle.

5. **Encouraging Collaborative Learning:** Collaborative learning opportunities allow students to work together, share ideas, and learn from one another. This approach not only enhances understanding but also builds essential communication and teamwork skills.

○ **E.g.** In a class of SS1 students

"In teams of three, investigate the pattern in the sequence of polygon interior-angle sums: 180º, 360º, 540º ... Derive the general formula $S_n = 180\,(n-2)$ and present your proof to the class."

Joint problem-solving and presentation tasks require each member to contribute, listen, and explain, building both content mastery and communication/teamwork skills.

6. **Adapting to Rapid Changes in Knowledge:** The field of mathematics, like all sciences, is continually evolving. Innovative pedagogies are needed to keep pace with new discoveries and ensure that the curriculum remains relevant and up-to-date.

○ **E.g.** During a class on compound interest,

"Using a spreadsheet (Microsoft Excel), model the compound-interest formula $A = P(1 + \frac{i}{n})^{nt}$

for principal P = ₦1000, annual rate i = 5%, over t = 3 years. Create columns for compounding frequency $n \in \{1,4,12\}$. Compute A in each row and plot a graph of A against n.

This activity connects secondary-level exponent knowledge to a real-world financial context, demonstrates how digital tools streamline repetitive calculations and prompts students to interpret and discuss how changing compounding frequency impacts returns – thereby modeling how mathematical practice evolves with technology.

7. **Addressing the Decline in Student Interest and Performance:** There has been a noticeable decline in students' interest in mathematics and their performance in this subject. Innovative pedagogical approaches can make mathematics more engaging and relevant, reversing these trends and inspiring more students to pursue STEM careers.

○ **E.g.** While teaching linear equations,

Task your class with designing the most cost-efficient rectangular garden that uses no more than 100m of fencing and encloses the maximum possible area. Have each group derive the function A(x) = x (50 – x). Find its maximum via completing the square, and present their optimal dimensions.

Framing the optimization as a tangible, real world challenge sparks engagement and demonstrates the practical relevance of algebraic techniques, reversing "math apathy".

Some of the Key Innovative Teaching Strategies in Secondary School Mathematics
1. Integration of Technology: This is grouped under interactive software and the Innovative use of Technology.

(a) Interactive Software: Tools like Desmos, Geogebra, and other educational software allow students to visualize and manipulate mathematical concepts. These software provide interactive simulations, visualizations, and activities to help students better understand mathematical concepts, explore mathematical relationships by manipulating variables and observing changes, and enable teachers to create custom learning activities and assessments.

Desmos is an educational platform that enhances the learning and teaching of mathematics in the classroom. It is mostly used to teach graphing, especially when changing a variable or term will cause the graph to shift or morph in any way. The biggest tool provided by Desmos is its graphing calculator, which can be used not just for plotting interactive graphs, but also can be manipulated in real-time. **https://www.desmos.com**

In Desmos, sliders are interactive tools that allow users to change the values of variables dynamically, adjusting a graph in real-time. Sliders are particularly useful for exploring the effects of different parameters in functions, such as those in quadratic equations. Sliders make Desmos a powerful, interactive tool for teaching math concepts, as they bring abstract equations to life through direct manipulation.

For example, when teaching about the quadratic graph, Desmos can be used to show the students the graphical representation of the most basic quadratic equation $y = x^2$, where there is no constant (so the vertex is at the origin), no coefficient of x (so the vertex of the parabola is centered on the y-axis) and the coefficient of x^2 (meaning that the Parabola is neither stretched nor compressed).

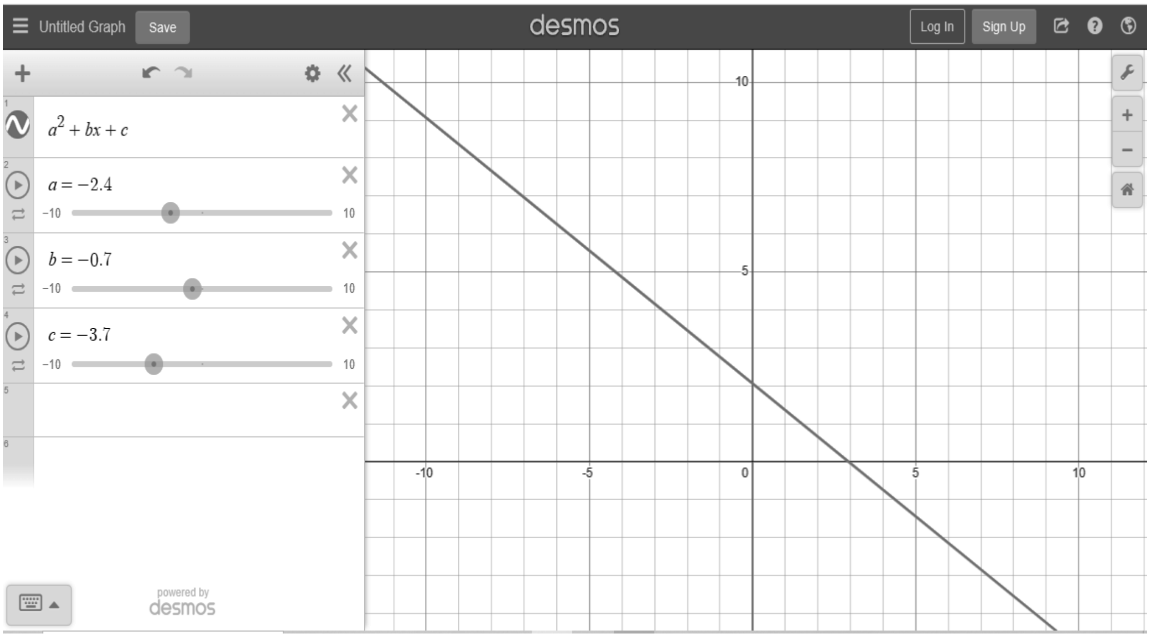
- Following this illustration, the teacher can then add a constant to the equation to show how that would affect the general parabola.
- Next, the teacher can add a coefficient for x and illustrate how that moves the vertex of the parabola.
- The teacher can also use this to teach and help the students visualize how to find the equation of a parabola, from its features, roots, and vertex.
- In the case of the Sine and the Cosine graph, Desmos can play a big role in helping students understand how the Sine graph works.
- For equation y = sin(x), Desmos can help illustrate the Sine wave and make its features clearer to the students.
- Amplitude is 1 because the co-efficient of sine is 1 so the graph oscillates between 1 and -1
- So, the teacher can also graph y = 2 sin (x) to show that the amplitude will become 2 and the oscillation will be between 2 and -2
- The period is 2 π so the wave completes a full cycle in 2 π or roughly, 6.28… on the x-axis
- So, the teacher can give x a coefficient of 2 to show how that will reduce the period by half, hence the graph completes a cycle in π and the same for phase shifts and so on.
- Another good thing about desmos is that it has templates of general math graphs saved, including;
- Line graph: slope-intercept, point-slope, two-point forms
- Parabolas: standard form, vertex form, standard + tangent
- Trigonometry: period and amplitude, phase, unit circle
- Transformations: translation, scaling, and inverse
- Statistics: linear regression
- Calculus: derivatives, integrals, etc.

To present Desmos practically, focus on hands-on steps and relatable examples, showing pre-service teachers exactly how they can use Desmos in the classroom for teaching quadratic functions. Here's a

Steps on the use of Desmos tools and features with specific, student-friendly activities.

1. Setting up a Basic Quadratic Function with Sliders

- Objective: Familiarize teachers with Desmos sliders to explore how changing values impact a quadratic graph.
- Start by typing the function ($y = ax^2 + bx + c$) directly into Desmos.
- Add sliders for (a), (b), and (c) by clicking on each variable; this lets teachers (and students) interactively change the values to see their effect.
- Explanation:
- Changing (a) controls the width and direction of the parabola.
- Modifying (b) affects the tilt or horizontal shift.
- Adjusting (c) moves the parabola up or down, setting the y-intercept.



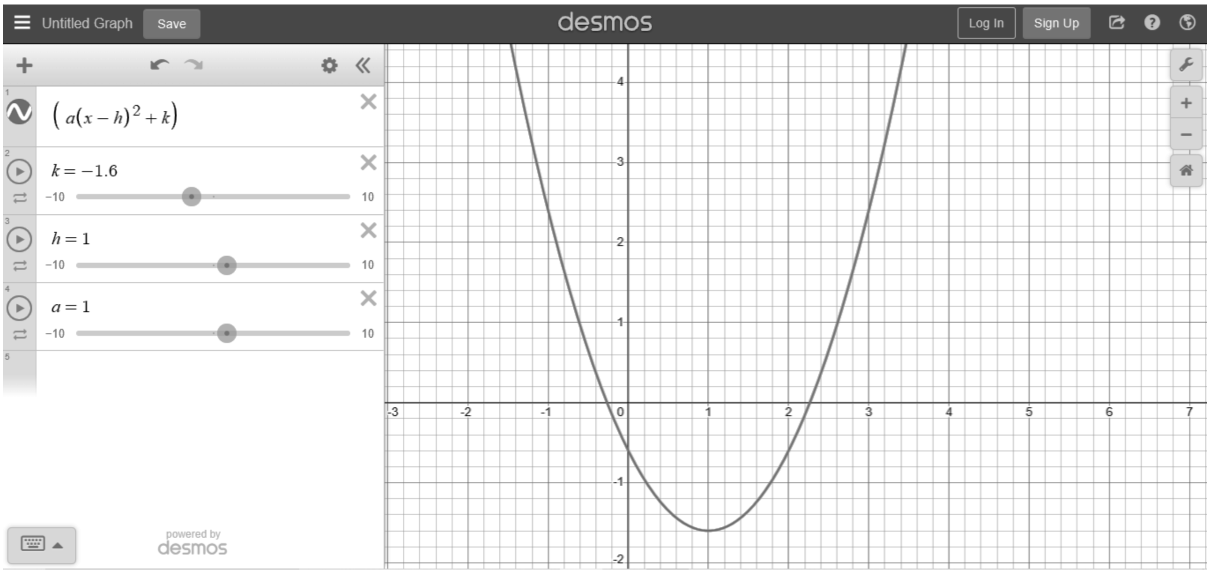
- **Practical Tip**: Encourage teachers to have students predict what will happen as each parameter changes before adjusting it with sliders, promoting active engagement and discovery.

2. Using Vertex Form for Easy Transformations

- Objective: Show how the vertex form ($y = a(x - h)^2 + k$) visually represents shifts and the parabola's vertex.

Example:

- Type ($y = a(x - h)^2 + k$) into Desmos and add sliders for (a), (h), and (k).
- Adjust (h) and (k) to move the vertex, and use (a \) to open the parabola up or down.



Emphasize that:
- (h) shifts the graph left or right.

- (k) moves the graph vertically, identifying the vertex as ((h, k)).

- Practical Tip: Have teachers ask students to adjust the sliders to match the graph of a given function, reinforcing how each part of the equation translates directly into a visual change.

3. Exploring X-Intercepts with Factored Form

- Objective: Use factored form ($y = a(x - r_1)(x - r_2)$) to explore x-intercepts and roots of quadratic equations.

Example:

- Input ($y = a(x - r_1)(x - r_2)$) and create sliders for (r_2), (r_2), and (a).
- Adjust (r_1) and (r_2) to see the x-intercepts change, showing how the parabola's roots (solutions) shift.

This helps students visually connect algebraic solutions to their graphical representations, showing them the x-intercepts are the points where the parabola crosses the x-axis.

- **Practical Tip**: (1) Challenge students to adjust the values so that the parabola touches the x-axis in only one place, introducing the concept of a double root. (2) Include reflection questions, such as "What did you notice about the graph when changing the slider?" This encourages students to articulate their understanding, deepening comprehension.

Through these activities, pre-service teachers can grasp Desmos's functionality in a classroom-ready, practical way, helping them create engaging and visual lessons on quadratic functions that build students' intuition and understanding.

GeoGebra

GeoGebra is a dynamic mathematics software that integrates various mathematical disciplines into a single platform, including geometry, algebra, spreadsheets, graphing, statistics, and calculus. It is designed for all levels of education, making it a versatile tool for both teaching and learning mathematics. Just like Desmos, GeoGebra's biggest feature is its graphing calculator, which can be used similarly.

•Both Desmos and GeoGebra allow for the plotting of graphs, GeoGebra has a selection of tools that allow you to annotate the graphs to certain degrees. GeoGebra has a feature that allows you to download your graph and illustrations made on it either as an image, which you can attach to another document or as a file on its own. This is very beneficial for planning instructional material.

Another amazing feature that GeoGebra has is the wide range of simulations of different math topics, available under "community resources." A great example can be teaching transversal using one of the simulation resources "alternating internal angles" alternating internal angles.

Specific Secondary School Mathematics Topics that can be taught with GeoGebra

1. **Geometry**: GeoGebra is particularly strong in teaching basic geometric concepts such as:
- Geometric Transformations: Students can explore reflections, rotations, translations, and dilations interactively, enhancing their understanding of these concepts through visualization and manipulation.
- Properties of Shapes: The software allows for the investigation of properties of various geometric figures, helping students understand the relationships and characteristics of shapes.

2. **Algebra**: GeoGebra aids in teaching algebraic concepts, including:
- Functions and Graphing: Students can visualize functions and their transformations, making it easier to grasp concepts like linear, quadratic, and exponential functions.
- Solving Equations: The platform supports the interactive exploration of algebraic equations, allowing students to see the effects of changing parameters in real-time.

3. **Calculus**: GeoGebra can be used to teach calculus topics such as:
- Limits and Continuity: Students can visualize the behavior of functions as they approach specific points.
- Derivatives and Integrals: The software allows for dynamic exploration of rates of change and areas under curves, making these concepts more tangible.

4. **Statistics and Probability**: GeoGebra supports the teaching of statistical concepts by:
- Data Visualization: Students can create and manipulate graphs and charts, facilitating a better understanding of data analysis and interpretation.
- Probability Distributions: The software allows for the exploration of different probability distributions, helping students visualize and understand concepts like normal distribution.

5. **Measurement and Units**: GeoGebra can enhance lessons on measurement by allowing students to:
- Explore Area and Volume: Students can manipulate shapes to understand how changes affect area and volume, reinforcing measurement concepts through hands-on interaction.

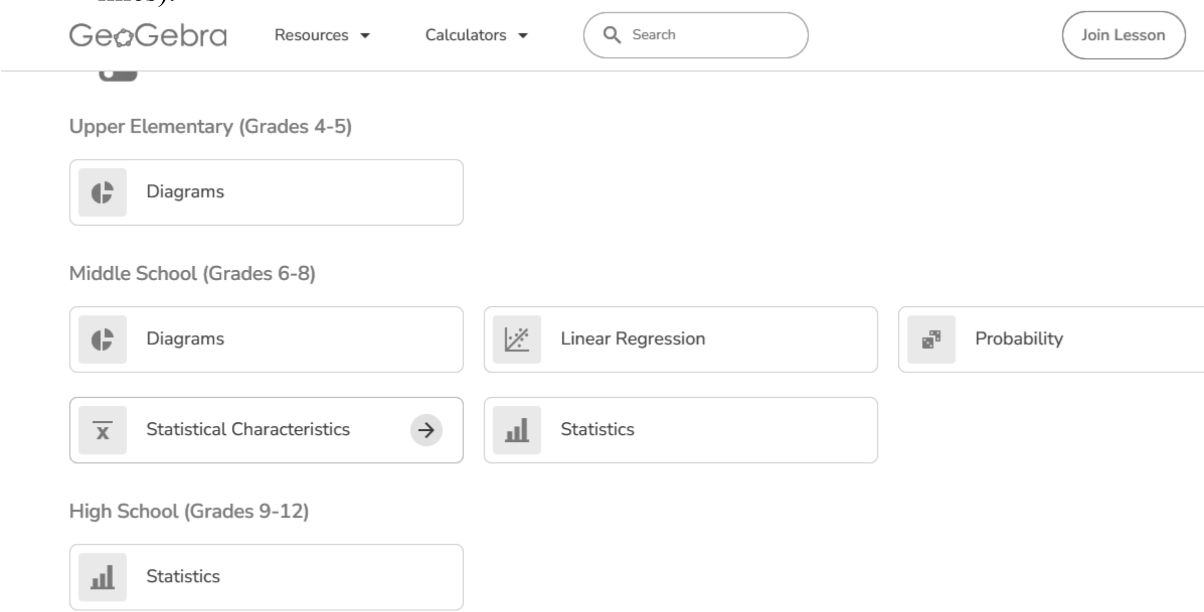
6. **Number Sense and Operations**: For younger students, GeoGebra provides tools to:
- Explore Basic Operations: Interactive tools help students understand addition, subtraction, multiplication, and division through visual aids and manipulatives.

To present the GeoGebra interface practically, it's helpful to guide pre-service teachers through an interactive tour, focusing on how they can directly use each feature in their teaching. Here's how you could break down the interface in a practical, classroom-friendly way:

A step-by-step guide on how to use GeoGebra

1. Toolbars and Basic Tools: Teach navigation and tool functions as they relate to common teaching tasks.

- Begin with a simple task, like constructing a triangle, to get familiar with these tools.
- Move Tool (hand icon): To adjust objects on the screen by dragging them around.
- Point Tool: Start by adding a point anywhere on the canvas. Discuss how points are the basis for most constructions (like shapes and lines).
- Line, Ray, and Segment Tools: Show how each of these is essential for geometry tasks like creating shapes or showing specific relationships (e.g., parallel and perpendicular lines).



Practical Tip: Show that holding the mouse over a tool reveals a description, and right-clicking gives additional options.

2. **Algebra and Graphics Views:** Explain how the Algebra and Graphics Views work together to provide a dynamic teaching experience.

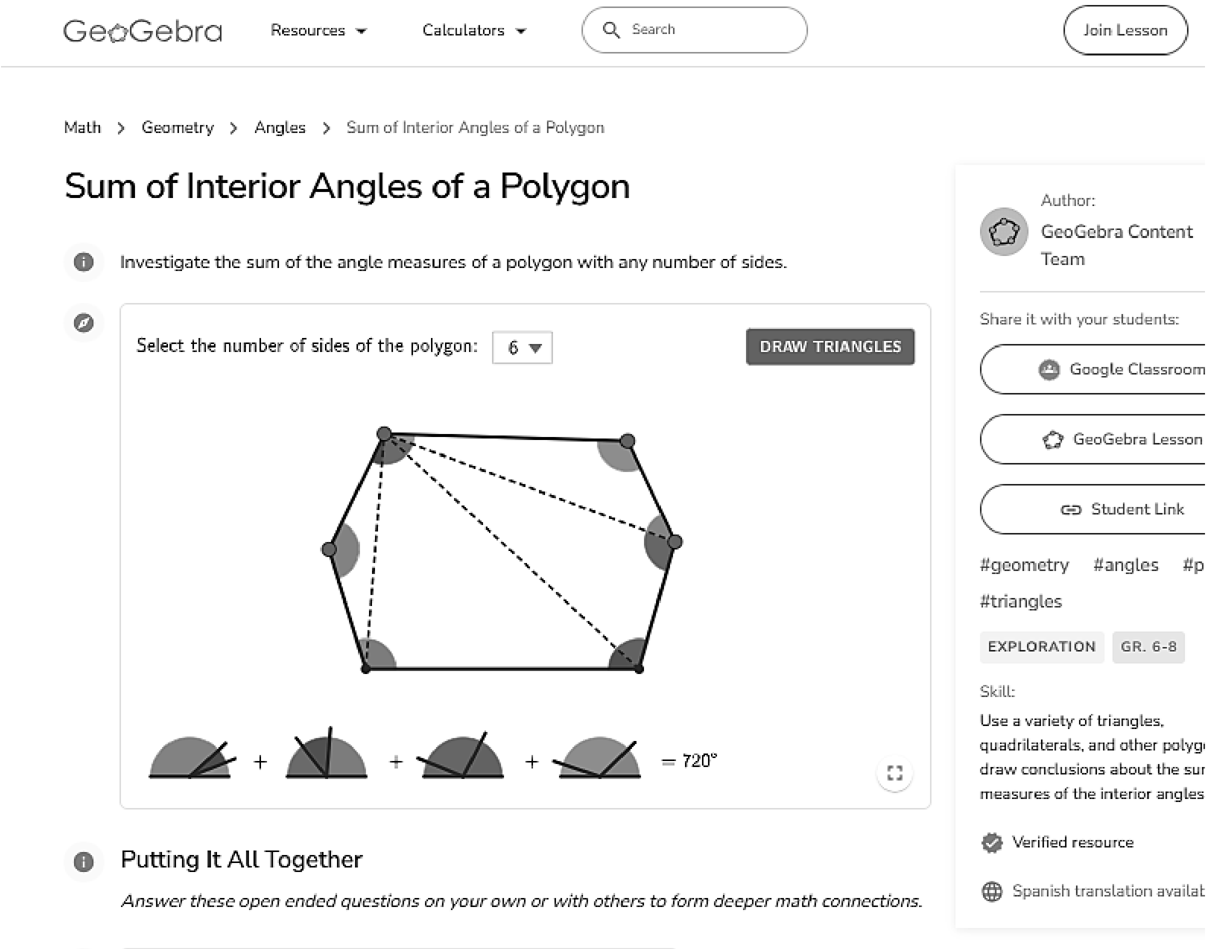
- Place a few points and construct a triangle, then observe how the algebraic coordinates and equations appear in the Algebra View.
- Algebra View: This left panel displays objects numerically (e.g., point coordinates, lengths, angles).
- Graphics View: The central canvas where constructions are visualized.

Practical Tip: Emphasize how changes in Graphics View reflect immediately in Algebra View, reinforcing the relationship between numeric properties and visual representations—this can deepen students' understanding of geometric properties.

3. **Construction Tools for Dynamic Geometry**: Enable dynamic, hands-on exploration with constructions that adjust in real time.

- Activity: Create a square and add a Slider tool to manipulate its side length dynamically.

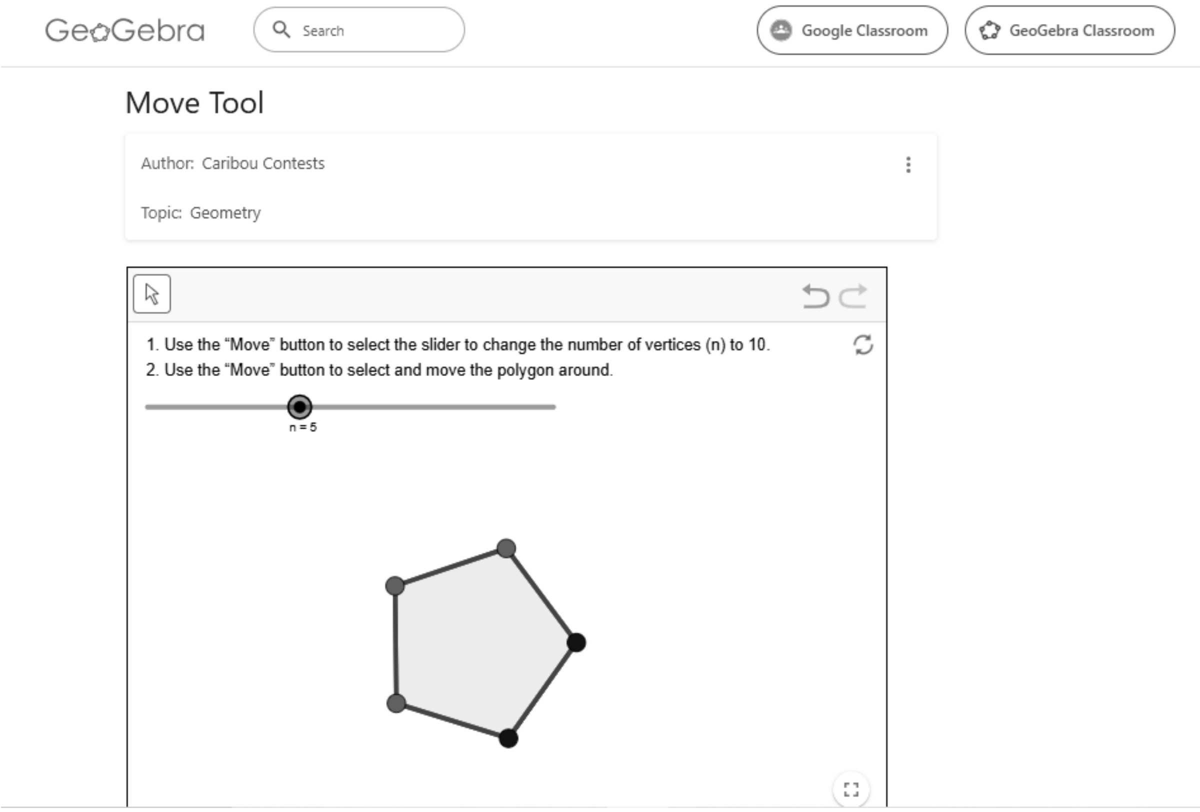
- Polygon Tool: Use this to construct shapes that maintain their properties when adjusted.
- Slider Tool: Useful for exploring variable dimensions (e.g., resizing shapes or adjusting angles). Sliders can help students experiment with concepts like similarity and proportionality without needing complex algebra.



4. Measurement and Calculation Tools: Demonstrate tools that calculate lengths, angles, and areas, bringing measurable insights to constructions.
   - **Activity**: Construct a triangle and use the angle and Distance or Length tools to measure its angles and side lengths.
   - **Angle Tool:** Measures the angle between three points. Discuss how this can help students verify geometric properties.
   - **Distance or Length Tool**: Useful for showing relationships, like the hypotenuse of a right triangle.

Practical Tip: These tools can verify or reinforce geometric theorems, such as angle sums or congruence rules.

5. Interactivity with the Move Tool: Emphasize how the Move Tool enables direct interaction, fostering exploration.
   - **Activity:** Construct a quadrilateral and ask teachers to drag vertices to see the effects.
   - **Dynamic Behavior**: As teachers move points, lines or angles update, illustrating geometric properties like side ratios or angles in real-time.



Practical Tip: This tool helps students test hypotheses visually, which is especially useful for learning about transformations and congruence.

6. **Worksheet Creation and Exportation**: Show teachers how to save, export, and share activities they create.

- **Activity**: Walk through saving a file and embedding it into an online platform or exporting as a worksheet.
- **Save & Export Options**: Teachers can save files for classroom use or export as interactive worksheets to upload on learning management systems.
- **GeoGebra Classroom**: Mention that they can use GeoGebra Classroom to assign and monitor activities in real-time. This feature allows for easy sharing and integration into digital learning, helping teachers create lasting, reusable resources.

Through these steps, pre-service teachers can focus on each feature practically, learning not just how to use GeoGebra but how to apply it in ways that make geometry interactive, accessible, and engaging for students.

2. Innovative Use of Technology: Interactive tools like Smart Boards should be utilized to engage students and enhance the learning experience.

Interactive Whiteboards (Smart Boards)

Interactive whiteboards (commonly known as Smart Boards) combine the functionality of a large touch-sensitive display with specialized software (e.g., SMART Notebook, Lumio) to deliver dynamic, multimedia-rich mathematics lessons. Projecting the computer desktop onto the board's surface lets you—and your students—manipulate graphs, diagrams, and images directly with fingers or pens. Multiple users can interact simultaneously, fostering collaboration, while built-in tools for drawing, highlighting, and saving annotations make it easy to illustrate concepts and preserve student work for review. By integrating videos, websites, and interactive applets, Smart Boards transform passive note-taking into an engaging, hands-on experience.

Sample Activity
Demonstrating the Pythagorean identity:

1. Display on the Smart Board the unit-circle diagram and the expressions $\tan^2 x$ and $\sec^2 x$.
2. Invite students to draw a right triangle inside the circle with hypotenuse 1, label the adjacent side cosx and the opposite side sinx.
3. Ask a volunteer to annotate the diagram by constructing lines representing $tanx = \frac{sinx}{cosx}$ and $secx = \frac{1}{cosx}$
4. Guide the class in writing $1 + \tan^2 x = \sec^2 x$ by squaring those ratios—students highlight each step directly on screen to build the proof together.

3. Collaborative Learning Approach

You will organize cooperative, team-based activities that encourage students to share ideas, challenge each other's thinking, and build deeper understanding through interaction.

Key Features of Collaborative Learning in Mathematics

1. **Student-Centered Learning**: Students take an active role in their learning process, rather than passively receiving information from the teacher. Learning responsibilities are also shared among group members, promoting accountability and mutual support.
   - **E.g.** In small groups, students explore the function $y = mx + c$ by each plotting a different line on a graph paper, then collectively determine how changing m or c shifts the graph
2. **Group Work and Interaction**: Groups are often heterogeneous, comprising students with different abilities, backgrounds, and perspectives to foster peer learning. Students discuss problems, explain their reasoning, and listen to others' ideas, which helps clarify their understanding.
   - **E.g.** Teams of students solve a system of linear equations by assigning one student to use substitution, one to use elimination, and one to verify both solutions graphically; then they compare methods.
3. **Problem-solving and Critical Thinking**: Tasks are designed to be challenging and open-ended, requiring collaborative effort and higher-order thinking skills. Students are encouraged to justify their solutions and critique the reasoning of others, promoting a deeper understanding of mathematical concepts.
   - **E.g.** Groups investigate the convergence of the series $\sum_{n=1}^{\infty} \frac{1}{n^p}$ for different p values using a spreadsheet, then present a proof sketch for why it diverges at $p \leq 1$
4. **Role of the Teacher**: The teacher acts as a facilitator, guiding students through the learning process, providing support, and intervening when necessary. Teachers provide scaffolds, such as hints or prompts, to support student learning and gradually remove them as students become more competent.

-**E.g.** Circulate while groups derive the quadratic formula, offering targeted hints (like "What happens if you complete the square here?") and then take a step back to let students apply the strategy independently.

Benefits of Collaborative Learning in Mathematics

Enhanced Understanding: Students gain a deeper understanding of mathematical concepts through discussion and explanation.

Improved Problem-Solving Skills: Collaborative learning encourages different approaches to problem-solving, improving students' ability to tackle complex problems.

Increased Engagement and Motivation: Working in groups can make learning more enjoyable and engaging, increasing student motivation.

Development of Social Skills: Students develop important social and communication skills, such as teamwork, leadership, and conflict resolution.

Strategies for Implementing Collaborative Learning

1. **Purposeful Group Formation**: Form groups based on diverse skill levels, and interests, or randomly to ensure a variety of interactions. Keep groups small (3-5 students) to ensure active participation and effective communication.
2. **Structured and Clearly Defined Tasks**: Design tasks with clear objectives and instructions to guide student collaboration. Assign roles (e.g., facilitator, recorder, checker) to ensure that each group member contributes and stays engaged.
3. **Assessment and Feedback**: Formative Assessment is implored, using ongoing assessment to monitor group progress and provide timely feedback. Peer and Self-Assessment are also incorporated to promote reflection and accountability.

Incorporating Collaborative Learning in a Quadratic Equation Class

Jigsaw Method: Divide the quadratic equation into parts (derivation, application, solving examples). Each group becomes an expert on one part. After mastering their section, they reassemble into new groups where each member teaches their part to the others.

- **Objective:** Understand and apply the quadratic formula.
- **Outcome**: Each student gains a comprehensive understanding of the quadratic formula through teaching and learning from peers.

Think-Pair-Share: Present a quadratic equation that can be factored. Students first think individually, then pair up to discuss/compare their solutions, and finally share their methods with the larger group or class.

- **Objective**: Learn to solve quadratic equations by factoring.
- **Outcome:** Discussion of different factoring techniques and clarification of common misconceptions.

Collaborative Problem Solving: Present a complex problem that cannot be factored easily. Each group is given a different quadratic equation to graph. They identify the vertex, axis of symmetry, and direction of the parabola. Groups present their graphs and discuss how changes in coefficients a, b, and c affect the graph's shape and position. Students work in groups to solve it, explaining their reasoning and considering different strategies.

- **Objective**: Learn to graph quadratic equations and understand the properties of parabolas.

- **Outcome:** Students develop a visual understanding of quadratic functions and their properties.

Online Collaboration Tools: Use interactive tools such as Desmos or GeoGebra to explore and visualize any given quadratic equations. Groups can collaboratively manipulate graphs and solve equations online, discussing their findings through platforms like Google Classroom or Microsoft Teams.

- **Objective**: Utilize technology to enhance collaboration and learning.
- **Outcome**: Enhanced engagement and understanding through interactive and visual learning.

In all these collaborative activities, the teacher should: (a) Provide clear instructions and expectations for each activity; (b) Actively monitor groups, offering guidance and support as needed; (c) Ensure all students are actively participating and contributing to the group work; develop fair and comprehensive assessment methods that account for both individual and group performance and also ensure sufficient resources (time, materials, and support) are available for effective collaborative learning.

4. Inquiry-Based Learning Approach

Inquiry-based learning (IBL) is an educational approach that emphasizes the student's role in the learning process, encouraging them to explore, ask questions, and discover new concepts for themselves. In teaching secondary school mathematics, IBL focuses on developing critical thinking, problem-solving skills, and a deeper understanding of mathematical concepts by engaging students in investigations and explorations.

Implementing inquiry-based learning (IBL) in secondary school mathematics involves creating an environment where students actively engage with mathematical concepts through questioning, exploration, and discovery.

Strategies for Implementing Inquiry-Based Learning in Secondary School Mathematics

1. **Create a Question**- Centered Classroom to Encourage Curiosity: Start lessons with intriguing questions or problems that spark curiosity. Allow students to pose their questions related to the topic at hand. Use student questions to guide class discussions and explorations.
2. **Design Open-Ended Tasks**: Provide tasks that have multiple entry points and various solution paths. Incorporate longer-term projects where students investigate a mathematical concept or real-world problem in depth. Use activities that encourage students to manipulate and explore mathematical ideas.
3. **Utilize Technology and Resources**: Integrate tools like GeoGebra or Desmos for visualizing and exploring mathematical concepts. Use videos, simulations, and interactive platforms that support inquiry-based learning.
4. **Foster a Collaborative Learning Environment**: Organize students into small groups to collaborate on problems and projects. Encourage students to explain their thinking to peers, fostering a deeper understanding. Hold regular whole-class discussions where students share their findings and reasoning.
5. **Scaffold the Inquiry Process**: Initially provide more structure and gradually release responsibility to the students. Use open-ended questions to guide students' thinking and

exploration. Supply students with tools, manipulative, and reference materials to aid their inquiry.

6. **Assess Understanding Formatively**: Provide ongoing, formative feedback to guide students' learning. Have students keep journals to reflect on their learning process and findings. Implement peer review and assessment to help students learn from each other.
7. **Cultivate a Growth Mindset**: Create a classroom culture where making mistakes is viewed as a valuable part of the learning process. Acknowledge and celebrate the effort and process of inquiry, not just the final answers.

Incorporating Inquiry-Based Learning in a Geometric Class

1. **Introduction to a Geometric Theorem:** Present a real-world problem or an intriguing scenario related to a geometric theorem. Example: "How can we prove that the angles in a triangle always add up to 180 degrees?"
2. **Encourage Hypothesis and Prediction:** Ask students to make predictions or hypotheses about the geometric theorem. Example: "What do you think happens to the angles in a triangle when you change its shape or size? Do they always add up to the same total?"
3. **Design and Conduct Investigations:** Have students work in groups to explore the theorem through hands-on activities and investigations. Use geometry tools like compasses, protractors, rulers, and dynamic geometry software (e.g., GeoGebra). Provide different types of triangles (scalene, isosceles, and equilateral) and ask students to measure and sum the angles. Example: "Measure the angles of various triangles and check if their sum is always 180 degrees. Try this with different types of triangles and see if the result holds."
4. **Explore Proof and Reasoning:** Guide students to develop a proof or reasoning for the geometric theorem. Encourage students to use their measurements and observations to create a formal proof of the theorem. Example: "Based on your measurements, can you construct a logical argument or proof to show that the angles in any triangle add up to 180 degrees?"
5. **Data Analysis and Interpretation:** Have students analyze their data, look for patterns, and interpret their results. Discuss the consistency of their findings and what they reveal about the theorem. Example: "Compare your results across different triangles. What patterns do you observe? How do these patterns support the theorem?"
6. **Discussion and Reflection**: Facilitate a class discussion where students share their findings, compare different proofs, and reflect on their learning process.

Sample Questions: "What challenges did you encounter while proving the theorem? How did different approaches compare? What did you learn from this activity?" "Discuss how your measurements and proofs relate to the theorem that the sum of the angles in a triangle is always 180 degrees."

ASSESSMENT

- Summarize the main pedagogical innovations discussed in this chapter.
- How do these innovations differ from traditional teaching methods?
  (The teacher should look for a clear and concise summary that highlights key innovations such as new teaching strategies or the use of technology. Compare these with traditional methods to show understanding).
- Use GeoGebra to solve a system of equations x^2 +2x + 8 graphically.

(The teacher should provide step-by-step hints or partial solutions to guide students through the problem-solving process. Use color coding to indicate correct and incorrect parts of their solutions.

Summary
This chapter explores various innovative teaching methods aimed at enhancing the learning experience and outcomes in secondary school mathematics. It emphasizes the shift from traditional teacher-centered approaches to more student-centered, interactive methods which include;

1. **Technology Integration**: Utilizing digital tools and resources like educational software, online platforms, and interactive whiteboards to facilitate better understanding and engagement with mathematical concepts.
2. **Inquiry-Based Learning**: Encouraging students to explore mathematical problems through questioning, investigation, and discovery, fostering a deeper understanding and critical thinking skills.
3. **Collaborative Learning**: Promoting teamwork and peer learning through group activities and discussions, helping students to learn from each other and develop social and communication skills. Implementing a collaborative learning approach in secondary school mathematics can create a dynamic and interactive learning environment, helping students to develop not only mathematical skills but also essential life skills such as teamwork, communication, and critical thinking.
4. **Active Learning Techniques**: Incorporating activities that engage students actively in the learning process, such as problem-solving sessions, group work, and hands-on projects.

References
Fennema, E., & Franke, K. (1992). Teachers' knowledge and its impact. In D. Grouws (Ed.), Handbook of research on mathematics teaching and learning (pp. 147-163). New York: Macmillan.

Gómez C., Bacelo M.I, Marbán, M. J., Palacios, A. (2023). Inquiry based mathematics education and attitudes towards mathematics: tracking profiles for teaching Mathematics Education Research Journal. https://doi.org/10.1007/s13394-023-00468-8

Further Reading
James W. Stigler and James Hiebert (2007). The Teaching Gap: Best Ideas from the World's Teachers for Improving Education in the Classroom" ISBN: 9781416586388, 1416586385. Publisher: Free Press

- This book explores various innovative teaching practices in mathematics education, comparing teaching methods from different countries.
  John Hattie, Douglas Fisher, and Nancy Frey (2016). "Visible Learning for Mathematics: What
  Works Best to Optimize Student Learning". Publisher: Corwin Press.
- This book provides research-based strategies and practices that can enhance mathematics teaching and learning in secondary schools.
- Sharma, M. (2015). "Innovative Pedagogies in Mathematics: Digital Technology Integration and Inquiry-Based Learning." International Journal of Mathematical Education in Science and Technology, 46(3), 371-389.