International Research Journal of Applied Sciences, Engineering and Technology
Vol.9, No. 06, June-2023;
ISSN (1573-1405);
p –ISSN 0920-5691
Impact factor: 6.52

SOLUTION OF A LARGE EVD COMPARTMENTAL MODEL SYSTEM USING HOMOTOPY PERTURBATION METHOD

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Abstract: Ebola Virus Disease is a disease (EVD) that is endemic for a long time now as we keep getting information continually of its recent activities in some parts of the world. The 2014 Ebola outbreak brought about the interest of researchers in the area of the disease. This present research presents a large eighteen compartments dynamic equation of Ebola Virus Disease with control measures. We applied the Homotopy Perturbation Method (HPM) technique to determine an approximate (analytic) solution of the model. HPM produced a solution that is close (converges) to the exact value. This work has been able to show that a large eighteen compartmental EVD model equation can be solved using HPM which has not been shown before in literature.

Key words: Approximate solution; Ebola Virus Disease Model; Homotopy Perturbation Method, Maple software.

Abbreviations; Ebola virus Disease (EVD), Homotopy Perturbation Method (HPM).

1. INTRODUCTION

Homotopy perturbation method (HPM) was designed by scientists and engineers to help in solving non-linear ordinary differential equation problems. This is because the Homotopy Perturbation method actually transforms a difficult non-linear problem that is hard to obtain its solution into a less difficult problem that can be solved. The HPM was first used by He in the year 1999, He (1999) and was improved in the year 2000 and 2006 respectively by He, He (2000a); He (2006). The HPM helps us to obtain an approximate analytic solution of a non-linear dynamic system. So many researchers have succeeded in obtaining analytical solution for different physical problems over the past years. Mathematical modelling of dynamic systems in infectious disease modelling using non-linear system of differential equations has drawn the attention of so many people over the years[14]. Epidemic model is simply a way of explaining the transmission dynamics of a communicable disease. Ebola virus disease (EVD) is a communicable infectious disease that mathematical model can be used to study. The disease stormed West Africa and did a lot of havoc during the Ebola outbreak in 2014(Ahman et al.; 2020). So many control measures such as contact tracing, quarantine, isolation and vaccine are now in use in the case of another outbreak (Ahman *et al.*, 2021). So many researchers in recent times have applied HPM in obtaining approximate solutions to infectious disease compartmental model dynamic system of equations. Omale D. and Gochhait S. (2018) applied HPM in obtaining the analytical solution of a HIV/AIDS compartmental model with a dynamic system of equation that is nine compartments. Atindiga *et al.*, (2020) applied HPM in obtaining the approximate solution of a non-linear system of differential equation of a disease model that is seven compartments. Didigwu *et al.*, (2020) applied HPM in solving an Ebola model dynamic system of equations with seven compartments and they all obtained an approximate solution without much difficulty. Many other researchers applied HPM in obtaining approximate solutions [2] [4] [5] [10] [15] but none have applied HPM in solving a system of EVD model equation that is eighteen compartments.

2. The Ebola Virus Disease Model Equation

Considering the following EVD model equation with control measures by Ahman *et al.*, (2021);

$$\left.\begin{aligned}
&(1)\ \frac{dS}{dt} = \Pi - (\mu + r + K_1X)S + \tau R\\
&(2)\ \frac{dS_v}{dt} = rS - (\mu + \lambda + K_2Z)S_v\\
&(3)\ \frac{dS_u}{dt} = K_1XS - (\mu + \sigma + K_3Y)S_u\\
&(4)\ \frac{dS_{vc}}{dt} = \lambda S_v - (\mu + \beta_1)S_{vc}\\
&(5)\ \frac{dS_{vn}}{dt} = K_2ZS_v - (\mu + \beta_2)S_{vn}\\
&(6)\ \frac{dS_{uc}}{dt} = \sigma S_u - (\mu + \beta_3)S_{uc}\\
&(7)\ \frac{dS_{un}}{dt} = K_3YS_u - (\mu + \beta_4)S_{un}\\
&(8)\ \frac{dE}{dt} = \beta_1S_{vc} + \beta_2S_{vn} + \beta_3S_{uc} + \beta_4S_{un} - (\mu + \alpha_1 + \alpha_2)E\\
&(9)\ \frac{dE_Q}{dt} = \alpha_1E - (\mu + \theta_1)E_Q\\
&(10)\ \frac{dE_T}{dt} = \alpha_2E - (\mu + \theta_2 + \rho)E_T\\
&(11)\ \frac{dI_T}{dt} = C_1E_Q + C_2E_T - (d_1 + J_2)I_T\\
&(12)\ \frac{dI_i}{dt} = C_3E_Q + C_4E_T - (d_2 + J_1)I_i\\
&(13)\ \frac{dI_N}{dt} = C_5E_Q + C_6E_T - d_3I_N\\
&(14)\ \frac{dR}{dt} = J_1I_i + J_2I_T + \rho E_T - \tau R\\
&(15)\ \frac{dD_u}{dt} = d_3I_N + d_2I_i + d_1I_T - qD_u\\
&(16)\ \frac{dS_r}{dt} = \Lambda - \mu_rS_r\\
&(17)\ \frac{dE_r}{dt} = \omega S_r - \mu_rE_r\\
&(18)\ \frac{dI_r}{dt} = \Phi E_r - d_4I_r
\end{aligned}\right\} \quad (2.1)$$

with the initial conditions; $S(0) = S_0$, $S_v(0) = S_{v0}$, $S_u(0) = S_{u0}$, $S_{vc}(0) = S_{vc0}$, $S_{vn}(0) = S_{vn0}$, $S_{uc}(0) = S_{uc0}$, $S_{un}(0) = S_{un0}$, $E(0) = E_0$, $E_Q(0) = E_{Q0}$, $E_T(0) = E_{T0}$, $I_T(0) = I_{T0}$, $I_i(0) = I_{i0}$, $I_N(0) = I_{N0}$, $R(0) = R_0$, $D_u(0) = D_{u0}$, $S_r(0) = S_{r0}$, $E_r(0) = E_{r0}$ *and* $I_r(0) = I_{r0}$.

we let; $B_1 = (\mu + r + K_1X), B_2 = (\mu + \lambda + K_2Z), B_3 = (\mu + \sigma + K_3Y), B_4 = (\mu + \beta_1), B_5 = (\mu + \beta_2), B_6 = (\mu + \beta_3), B_7 = (\mu + \beta_4) B_8 = (\mu + \alpha_1 + \alpha_2), B_9 = (\mu + \theta_1), B_{10} = (\mu + \theta_2 + \rho), B_{11} = (d_1 + J_2), B_{12} = (d_2 + J_1)$
So that after substitutions equation (2.1) becomes;

$$\left.\begin{aligned}
&(1)\ \frac{dS}{dt} = \Pi - B_1S + \tau R\\
&(2)\ \frac{dS_v}{dt} = rS - B_2S_v\\
&(3)\ \frac{dS_u}{dt} = K_1XS - B_3S_u\\
&(4)\ \frac{dS_{vc}}{dt} = \lambda S_v - B_4S_{vc}\\
&(5)\ \frac{dS_{vn}}{dt} = K_2ZS_v - B_5S_{vn}\\
&(6)\ \frac{dS_{uc}}{dt} = \sigma S_u - B_6S_{uc}\\
&(7)\ \frac{dS_{un}}{dt} = K_3YS_u - B_7S_{un}\\
&(8)\ \frac{dE}{dt} = \beta_1S_{vc} + \beta_2S_{vn} + \beta_3S_{uc} + \beta_4S_{un} - B_8E\\
&(9)\ \frac{dE_Q}{dt} = \alpha_1E - B_9E_Q\\
&(10)\ \frac{dE_T}{dt} = \alpha_2E - B_{10}E_T\\
&(11)\ \frac{dI_T}{dt} = C_1E_Q + C_2E_T - B_{11}I_T\\
&(12)\ \frac{dI_i}{dt} = C_3E_Q + C_4E_T - B_{12}I_i\\
&(13)\ \frac{dI_N}{dt} = C_5E_Q + C_6E_T - d_3I_N\\
&(14)\ \frac{dR}{dt} = J_1I_i + J_2I_T + \rho E_T - \tau R\\
&(15)\ \frac{dD_u}{dt} = d_3I_N + d_2I_i + d_1I_T - qD_u\\
&(16)\ \frac{dS_r}{dt} = \Lambda - \mu_rS_r\\
&(17)\ \frac{dE_r}{dt} = \omega S_r - \mu_rE_r\\
&(18)\ \frac{dI_r}{dt} = \phi E_r - d_4I_r
\end{aligned}\right\} \quad (2.2)$$

3 BASIC IDEA OF HE'S HOMOTOPY PERTURBATION METHOD

To show the basic idea of He's homotopy perturbation method; we consider the non linear differential equation

$A(u) - d(r) = 0 \quad r \in \Omega$ (3.1)

Subject to the boundary condition of:

$B\left(u, \frac{\partial u}{\partial n}\right) = 0, r \in \Gamma$ (3.2)

Where; A is the general differential operator and B is the boundary operator

$d(r)$; a known analytical solution and Γ is the boundary of the domain Ω ,

The general operator, A can be divided into two parts namely; L and N where L is linear part and N is the non linear part. Hence (3) can be written as;

$L(u) + N(u) - d(r) = 0, \quad r \in \Omega$ (3.3)

We then construct a homotopy $V(r, p)$ such that

$V(r, p): \Omega \times [0,1] \to R$ This satisfies

$H(r, p) = (1 - p)[L(v) - L(u_0)] + p[L(v) + N(v) - d(r)] = 0$ (3.4)

$P \in [0,1], \quad r \in \Omega$

$H(r, p) = L(v) - L(u_0) + pL(u_0) + [N(v) - d(r)] = 0$ (3.5)

Where; $L(u)$ is the linear part and $N(u)$ is the non-linear part

$L(u) = L(v) - L(u_0) + pL(u_0)$ and $N(u) = pN(v)$

$P \in [0,1]$ is an embedding parameter, while u_0 is an initial approximation of equation (3.1) which satisfies the boundary conditions.

Certainly, when considering equations (3.4) and (3.5), we have

$H(v, 0) = L(v) - L(u_0) = 0$ (3.6)

$H(v, 1) = A(v) - d(r) = 0$ (3.7)

The changing process of p from zero to unity is just that of $V(r, p)$ from u_0 to $u(r)$. In topology, this is called deformation while $L(v) - L(u_0)$, $A(v) - d(r)$ are called homotopy.

According to Homotopy perturbation method (HPM), we can first use the embedding parameter, p as a small parameter and assume solution for equation (3.4) and (3.5) which can be expressed as;

$V = v_0 + pv_1 + p^2 v_2 + \cdots$ (3.8)

when we let $p = 1$, we will obtain an approximate solution of equation (3.8) as

$U = \lim_{p \to 1} v = v_0 + v_1 + v_2 + \cdots$ (3.9)

Equation (3.9) is the analytical solution of (3.1) using homotopy perturbation method.

For the convergence of (3.9), He (2003) and He (2006) made the following suggestions;

(i) The second derivative of $N(v)$ with respect to V must be small because the parameter p must be relatively large (i.e. $p \to 1$)

(ii) The norm of $L^{-1} \frac{\partial N}{\partial V}$ must be smaller than one to let the series converge.

4 Applying HPM to Obtain the Solution of the EVD Model

In this section we apply HPM to equation (2.2) by assuming the solution of the equation to be;

$$\left.\begin{array}{l}(1)\ S = s_0 + Ps_1 + P^2 s_2 + \cdots \\ (2)\ S_v = s_{v0} + Ps_{v1} + P^2 s_{v2} + \cdots \\ (3)\ S_u = s_{u0} + Ps_{u1} + P^2 s_{u2} + \cdots \\ (4)\ S_{vc} = s_{vc0} + Ps_{vc1} + P^2 s_{vc2} + \cdots \\ (5)\ S_{vn} = s_{vn0} + Ps_{vn1} + P^2 s_{vn2} + \cdots \\ (6)\ S_{uc} = s_{uc0} + Ps_{uc1} + P^2 s_{uc2} + \cdots \\ (7)\ S_{un} = s_{un0} + Ps_{un1} + P^2 s_{un2} + \cdots \\ (8)\ E = e_0 + Pe_1 + P^2 e_2 + \cdots \\ (9)\ E_Q = e_{Q0} + Pe_{Q1} + P^2 e_{Q2} + \cdots \\ (10)\ E_T = e_{T0} + Pe_{T1} + P^2 e_{T2} + \cdots \\ (11)\ I_T = i_{T0} + Pi_{T1} + P^2 i_{T2} + \cdots \\ (12)\ I_i = i_{i0} + Pi_{i1} + P^2 i_{i2} + \cdots \\ (13)\ I_N = i_{N0} + Pi_{N1} + P^2 i_{N2} + \cdots \\ (14)\ R = r_0 + Pr_1 + P^2 r_2 + \cdots \\ (15)\ D_u = d_{u0} + Pd_{u1} + P^2 d_{u2} + \cdots \\ (16)\ S_r = s_{r0} + Ps_{r1} + P^2 s_{r2} + \cdots \\ (17)\ E_r = e_{r0} + Pe_{r1} + P^2 e_{r2} + \cdots \\ (18)\ I_r = i_{r0} + Pi_{r1} + P^2 i_{r2} + \cdots \end{array}\right\} \quad (4.1)$$

From (1) of equation (2.2) we have;

$$\frac{dS}{dt} = \Pi - B_1 S + \tau R$$

The linear part is $\frac{dS}{dt} = 0$ and the non-linear part is $\Pi - B_1 S + \tau R = 0$

When we apply HPM, we have;

$$(1-P)\frac{dS}{dt} + P\left[\frac{dS}{dt} - (\Pi - B_1 S + \tau R)\right] = 0 \qquad (4.2)$$

Expanding (4.2) we have;

$$\frac{dS}{dt} - P\frac{dS}{dt} + P\frac{dS}{dt} - P(\Pi - B_1 S + \tau R) = 0, \qquad \Rightarrow \frac{dS}{dt} - P(\Pi - B_1 S + \tau R) = 0$$

$$\Rightarrow \frac{dS}{dt} - P\Pi + PB_1 S - P\tau R = 0 \qquad (4.3)$$

Substituting (1) and (14) of (4.1) into (4.3), we have

$$(s_0' + Ps_1' + P^2 s_2' + \cdots) - P\Pi + PB_1(s_0 + Ps_1 + P^2 s_2 + \cdots) - P\tau(r_0 + Pr_1 + P^2 r_2 + \cdots) = 0$$

Thus,

$$s_0' + Ps_1' + P^2 s_2' + \cdots - P\Pi + (PB_1 s_0 + P^2 B_1 s_1 + P^3 B_1 s_2 + \cdots) - (P\tau r_0 + P^2 \tau r_1 + P^3 \tau r_2 + \cdots) = 0$$

Now collecting the coefficient of powers of P, we have;

$$\left.\begin{array}{r} P^0: s_0' = 0 \\ P^1: s_1' - \Pi + B_1 s_0 - \tau r_0 = 0 \\ P^2: s_2' + B_1 s_1 - \tau r_1 = 0 \end{array}\right\} \quad (4.4)$$

Similarly, we apply HPM to (2) to (18) of (2.2) to obtain the following;

$$\left.\begin{array}{c} P^0: s'_{v0} = 0 \\ P^1: s'_{v1} - \mathrm{r}s_0 + \mathrm{B}_2 s_{v0} = 0 \\ P^2: s'_{v2} - \mathrm{r}s_1 + \mathrm{B}_2 s_{v1} = 0 \end{array}\right\} \quad (4.5)$$

$$\left.\begin{array}{c} P^0: s'_{u0} = 0 \\ P^1: s'_{u1} - K_1 X s_0 + \mathrm{B}_3 s_{u0} = 0 \\ P^2: s'_{u2} - K_1 X s_1 + \mathrm{B}_3 s_{u1} = 0 \end{array}\right\} \quad (4.6)$$

$$\left.\begin{array}{c} P^0: s'_{vc0} = 0 \\ P^1: s'_{vc1} - \lambda s_{v0} + \mathrm{B}_4 s_{vc0} = 0 \\ P^2: s'_{vc2} - \lambda s_{v1} + \mathrm{B}_4 s_{vc1} = 0 \end{array}\right\} \quad (4.7)$$

$$\left.\begin{array}{c} P^0: s'_{vn0} = 0 \\ P^1: s'_{vn1} - K_2 Z s_{v0} + \mathrm{B}_5 s_{vn0} = 0 \\ P^2: s'_{vc2} - K_2 Z s_{v1} + \mathrm{B}_5 s_{vn1} = 0 \end{array}\right\} \quad (4.8)$$

$$\left.\begin{array}{c} P^0: s'_{uc0} = 0 \\ P^1: s'_{uc1} - \sigma s_{u0} + \mathrm{B}_6 s_{uc0} = 0 \\ P^2: s'_{uc2} - \sigma s_{u1} + \mathrm{B}_6 s_{uc1} = 0 \end{array}\right\} \quad (4.9)$$

$$\left.\begin{array}{c} P^0: s'_{un0} = 0 \\ P^1: s'_{un1} - K_3 Y s_{u0} + \mathrm{B}_7 s_{un0} = 0 \\ P^2: s'_{un2} - K_3 Y s_{u1} + \mathrm{B}_7 s_{un1} = 0 \end{array}\right\} \quad (4.10)$$

$$\left.\begin{array}{c} P^0: e'_0 = 0 \\ P^1: e'_1 - \beta_1 s_{vc0} - \beta_2 s_{vn0} - \beta_3 s_{uc0} - \beta_4 s_{un0} + \mathrm{B}_8 e_0 = 0 \\ P^2: e'_2 - \beta_1 s_{vc1} - \beta_2 s_{vn1} - \beta_3 s_{uc1} - \beta_4 s_{un1} + \mathrm{B}_8 e_1 = 0 \end{array}\right\} \quad (4.11)$$

$$\left.\begin{array}{c} P^0: {e_Q}'_0 = 0 \\ P^1: {e_Q}'_1 - \alpha_1 e_0 + \mathrm{B}_9 e_{Q0} = 0 \\ P^2: {e_Q}'_2 - \alpha_1 e_1 + \mathrm{B}_9 e_{Q1} = 0 \end{array}\right\} \quad (4.12)$$

$$\left.\begin{array}{c} P^0: {e_T}'_0 = 0 \\ P^1: {e_T}'_1 - \alpha_2 e_0 + \mathrm{B}_{10} e_{T0} = 0 \\ P^2: {e_T}'_2 - \alpha_2 e_1 + \mathrm{B}_{10} e_{T1} = 0 \end{array}\right\} \quad (4.13)$$

$$\left.\begin{array}{c} P^0: {i_T}'_0 = 0 \\ P^1: {i_T}'_1 - \mathrm{C}_1 e_{Q0} - \mathrm{C}_2 e_{T0} + \mathrm{B}_{11} i_{T0} = 0 \\ P^2: {i_T}'_2 - \mathrm{C}_1 e_{Q1} - \mathrm{C}_2 e_{T1} + \mathrm{B}_{11} i_{T1} = 0 \end{array}\right\} \quad (4.14)$$

$$\left.\begin{array}{c} P^0: {i_i}'_0 = 0 \\ P^1: {i_i}'_1 - \mathrm{C}_3 e_{Q0} - \mathrm{C}_4 e_{T0} + \mathrm{B}_{12} i_{i0} = 0 \\ P^2: {i_i}'_2 - \mathrm{C}_3 e_{Q1} - \mathrm{C}_4 e_{T1} + \mathrm{B}_{12} i_{i1} = 0 \end{array}\right\} \quad (4.15)$$

$$\left.\begin{array}{r} P^0: i_{N_0}' = 0 \\ P^1: i_{N_1}' - C_5 e_{Q0} - C_6 e_{T0} + d_3 i_{N0} = 0 \\ P^2: i_{N_2}' - C_5 e_{Q1} - C_6 e_{T1} + d_3 i_{N1} = 0 \end{array}\right\} \quad (4.16)$$

$$\left.\begin{array}{r} P^0: r_0' = 0 \\ P^1: r_1' - J_1 i_{i0} - J_2 i_{T0} - \rho e_{T0} + \tau r_0 = 0 \\ P^2: r_2' - J_1 i_{i1} - J_2 i_{T1} - \rho e_{T1} + \tau r_1 = 0 \end{array}\right\} \quad (4.17)$$

$$\left.\begin{array}{r} P^0: d_{u0}' = 0 \\ P^1: d_{u1}' - d_3 i_{N0} - d_2 i_{i0} - d_1 i_{T0} + qD_{u0} = 0 \\ P^2: d_{u2}' - d_3 i_{N1} - d_2 i_{i1} - d_1 i_{T1} + qD_{u1} = 0 \end{array}\right\} \quad (4.18)$$

$$\left.\begin{array}{r} P^0: s_{r0}' = 0 \\ P^1: s_{r1}' - \Lambda + \mu_r s_{r0} = 0 \\ P^2: s_{r2}' + \mu_r s_{r1} = 0 \end{array}\right\} \quad (4.19)$$

$$\left.\begin{array}{r} P^0: e_{r0}' = 0 \\ P^1: e_{r1}' - \omega s_{r0} + \mu_r e_{r0} = 0 \\ P^2: e_{r2}' - \omega s_{r1} + \mu_r e_{r1} = 0 \end{array}\right\} \quad (4.20)$$

$$\left.\begin{array}{r} P^0: i_{r0}' = 0 \\ P^1: i_{r1}' - \phi e_{r0} + d_4 i_{r0} = 0 \\ P^2: i_{r2}' - \phi e_{r1} + d_4 i_{r1} = 0 \end{array}\right\} \quad (4.21)$$

From the first equation of (4.4), we have;

$s_0' = 0$, integrating, we obtain;

$s_0 = k_1$, where k_1 is the constant of integration

Applying the initial condition $S(0) = S_0$ we obtain

$k_1 = S_0$

$$\Rightarrow s_0 = S_0 \quad (4.22)$$

Similarly from the first equations of (4.5) to (4.21) we obtain the following;

$$\left.\begin{array}{c} s_{v0} = S_{v0}, s_{u0} = S_{u0}, s_{vc0} = S_{vc0}, s_{vn0} = S_{vn0}, \\ s_{uc0} = S_{uc0}, s_{un0} = S_{un0}, e_0 = E_0, i_{r0} = I_{r0}, \\ e_{Q0} = E_{Q0}, e_{T0} = E_{T0}, i_{T0} = I_{T0}, i_{i0} = I_{i0}, \\ i_{N0} = I_{N0}, r_0 = R_0, d_{u0} = D_{u0}, \\ s_{r0} = S_{r0}, e_{r0} = E_{r0}. \end{array}\right\} \quad (4.23)$$

From the second equation of (4.4)we have,

$s_1' - \Pi + B_1 s_0 - \tau r_0 = 0,$

$s_1' = \Pi - B_1 s_0 + \tau r_0$

$\Rightarrow \frac{ds_1}{dt} = \Pi - B_1 s_0 + \tau r_0$

$$\Rightarrow ds_1 = (\Pi - B_1 s_0 + \tau r_0)dt \quad (4.24)$$

Substituting $s_0$and r_0 into (4.24), we obtain;

$ds_1 = (\Pi - B_1S_0 + \tau R_0)dt$
Integrating with respect to t, we have;
$s_1 = (\Pi - B_1S_0 + \tau R_0)t + k_3$ (Where k_3 is constant of integration).
Applying the initial condition, we have;
$s_1(0) = 0, \ \Rightarrow k_3 = 0$
$\therefore s_1 = (\Pi - B_1S_0 + \tau R_0)t \qquad (4.25)$
Similarly, from the second equations of (4.5) to (4.21) we obtain the following;

$$\left.\begin{aligned}
s_{v1} &= (\mathrm{r}S_0 - \mathrm{B}_2S_{v0})t,\\
s_{u1} &= (K_1XS_0 - \mathrm{B}_3S_{u0})t,\\
s_{vc1} &= (\lambda S_{v0} - \mathrm{B}_4S_{vc0})t,\\
s_{vn1} &= (K_2ZS_{v0} - \mathrm{B}_5S_{vn0})t,\\
s_{uc1} &= (\sigma S_{u0} - \mathrm{B}_6S_{uc0})t,\\
s_{un1} &= (K_3YS_{u0} - \mathrm{B}_7S_{un0})t,\\
e_1 &= (\beta_1S_{vc0} + \beta_2S_{vn0} + \beta_3S_{uc0} + \beta_4S_{un0} - \mathrm{B}_8E_0)t,\\
e_{Q1} &= \left(\alpha_1E_0 - \mathrm{B}_9E_{Q0}\right)t,\\
e_{T1} &= (\alpha_2E_0 - \mathrm{B}_{10}E_{T0})t,\\
i_{T1} &= \left(\mathrm{C}_1E_{Q0} + \mathrm{C}_2E_{T0} - \mathrm{B}_{11}I_{T0}\right)t,\\
i_{i1} &= \left(\mathrm{C}_3E_{Q0} + \mathrm{C}_4E_{T0} - \mathrm{B}_{12}I_{i0}\right)t,\\
i_{N1} &= \left(\mathrm{C}_5E_{Q0} + \mathrm{C}_6E_{T0} - \mathrm{d}_3I_{N0}\right)t,\\
r_1 &= (\mathrm{J}_1I_{i0} + \mathrm{J}_2I_{T0} + \rho E_{T0} - \tau R_0)t,\\
d_{u1} &= (\mathrm{d}_3I_{N0} + \mathrm{d}_2I_{i0} + d_1I_{T0} - qD_{u0})t,\\
s_{r1} &= (\Lambda - \mu_{\mathrm{r}}S_{r0})t,\\
e_{r1} &= (\omega S_{r0} - \mu_{\mathrm{r}}E_{r0})t,\\
i_{r1} &= (\phi E_{r0} - \mathrm{d}_4I_{r0})t
\end{aligned}\right\} \quad (4.26)$$

From the third equation of (4.4), we have;
$s_2' + B_1s_1 - \tau r_1 = 0$
$s_2' = \tau r_1 - B_1s_1$
$\Rightarrow \frac{ds_2}{dt} = \tau r_1 - B_1s_1$
$\Rightarrow ds_2 = (\tau r_1 - B_1s_1)dt \qquad (4.27)$
Substituting r_1 and s_1 into (4.27) we obtain;
$ds_2 = [\tau(\mathrm{J}_1I_{i0} + \mathrm{J}_2I_{T0} + \rho E_{T0} - \tau R_0)t - B_1(\Pi - B_1S_0 + \tau R_0)t\,]dt$
$ds_2 = [\tau(\mathrm{J}_1I_{i0} + \mathrm{J}_2I_{T0} + \rho E_{T0} - \tau R_0) - B_1(\Pi - B_1S_0 + \tau R_0)\,]tdt$
Integrating with respect to t, we have;
$s_2 = [\tau(\mathrm{J}_1I_{i0} + \mathrm{J}_2I_{T0} + \rho E_{T0} - \tau R_0) - B_1(\Pi - B_1S_0 + \tau R_0)\,]\frac{t^2}{2} + k_5$
Applying the initial condition to obtain k_5 (the constant of integration), we have;
$s_2(0) = 0, \ \Rightarrow k_5 = 0$

$$\therefore s_2 = [\tau(J_1 I_{i0} + J_2 I_{T0} + \rho E_{T0} - \tau R_0) - B_1(\Pi - B_1 S_0 + \tau R_0)]\frac{t^2}{2} \quad (4.28)$$

Similarly, from the third equations of (4.5) to (4.21) we have the following;

$$\left.\begin{aligned}
s_{v2} &= [r(\Pi - B_1 S_0 + \tau R_0) - B_2(rS_0 - B_2 S_{v0})]\frac{t^2}{2},\\
s_{vc2} &= [\lambda(rS_0 - B_2 S_{v0}) - B_4(\lambda S_{v0} - B_4 S_{vc0})]\frac{t^2}{2},\\
s_{vn2} &= [K_2 Z(rS_0 - B_2 S_{v0}) - B_5(K_2 Z S_{v0} - B_5 S_{vn0})]\frac{t^2}{2},\\
s_{uc2} &= [\sigma(K_1 X S_0 - B_3 S_{u0}) - B_6(\sigma S_{u0} - B_6 S_{uc0})]\frac{t^2}{2},\\
s_{un2} &= [K_3 Y(K_1 X S_0 - B_3 S_{u0}) - B_7(K_3 Y S_{u0} - B_7 S_{un0})]\frac{t^2}{2},\\
e_2 &= [\beta_1(\lambda S_{v0} - B_4 S_{vc0}) + \beta_2(K_2 Z S_{v0} - B_5 S_{vn0}) + \beta_3(\sigma S_{u0} - B_6 S_{uc0}) +\\
&\quad \beta_4(K_3 Y S_{u0} - B_7 S_{un0}) - B_8(\beta_1 S_{vc0} + \beta_2 S_{vn0} + \beta_3 S_{uc0} + \beta_4 S_{un0} - B_8 E_0)]\frac{t^2}{2}\\
e_{Q2} &= [\alpha_1(\beta_1 S_{vc0} + \beta_2 S_{vn0} + \beta_3 S_{uc0} + \beta_4 S_{un0} - B_8 E_0) - B_9(\alpha_1 E_0 - B_9 E_{Q0})]\frac{t^2}{2}\\
e_{T2} &= [\alpha_2(\beta_1 S_{vc0} + \beta_2 S_{vn0} + \beta_3 S_{uc0} + \beta_4 S_{un0} - B_8 E_0) - B_{10}(\alpha_2 E_0 - B_{10} E_{T0})]\frac{t^2}{2}\\
i_{T2} &= [C_1(\alpha_1 E_0 - B_9 E_{Q0}) + C_2(\alpha_2 E_0 - B_{10} E_{T0}) - B_{11}(C_1 E_{Q0} + C_2 E_{T0} - B_{11} I_{T0})]\frac{t^2}{2}\\
i_{i2} &= [C_3(\alpha_1 E_0 - B_9 E_{Q0}) + C_4(\alpha_2 E_0 - B_{10} E_{T0}) - B_{12}(C_3 E_{Q0} + C_4 E_{T0} - B_{12} I_{i0})]\frac{t^2}{2}\\
i_{N2} &= [C_5(\alpha_1 E_0 - B_9 E_{Q0}) + C_6(\alpha_2 E_0 - B_{10} E_{T0}) - d_3(C_5 E_{Q0} + C_6 E_{T0} - d_3 I_{N0})]\frac{t^2}{2}\\
r_2 &= [J_1(C_3 E_{Q0} + C_4 E_{T0} - B_{12} I_{i0}) + J_2(C_1 E_{Q0} + C_2 E_{T0} - B_{11} I_{T0}) + \rho(\alpha_2 E_0 - B_{10} E_{T0}) -\\
&\quad \tau(J_1 I_{i0} + J_2 I_{T0} + \rho E_{T0} - \tau R_0)]\frac{t^2}{2}\\
d_{u2} &= [d_3(C_5 E_{Q0} + C_6 E_{T0} - d_3 I_{N0}) + d_2(C_3 E_{Q0} + C_4 E_{T0} - B_{12} i_{i0}) +\\
&\quad (C_1 E_{Q0} + C_2 E_{T0} - B_{11} I_{T0}) - q(d_3 I_{N0} + d_2 I_{i0} + d_1 I_{T0} - q(d_3 I_{N0} + d_2 I_{i0} + d_1 I_{T0} - qD_{u0}))]\frac{t^2}{2}\\
s_{r2} &= [-\mu_r(\Lambda - \mu_r S_{r0})]\frac{t^2}{2}\\
e_{r2} &= [\omega(\Lambda - \mu_r S_{r0}) - \mu_r(\omega S_{r0} - \mu_r E_{r0})]\frac{t^2}{2}\\
i_{r2} &= [\phi(\omega S_{r0} - \mu_r E_{r0}) + d_4(\phi E_{r0} - d_4 I_{r0})]\frac{t^2}{2}
\end{aligned}\right\} \quad (4.29)$$

Substituting s_0 , s_1 and s_2 into (1) of equation (4.1) we have;

$$S(t) = S_0 + P(P' - B_1 S_0 + \tau R_0)t + P^2[\tau(J_1 I_{i0} + J_2 I_{T0} + \rho E_{T0} - \tau R_0) - B_1(P' - B_1 S_0 + \tau R_0)]\frac{t^2}{2} + \cdots \quad (4.30)$$

Setting P = 1 we obtain;

$$S(t) = S_0 + (P' - B_1 S_0 + \tau R_0)t + [\tau(J_1 I_{i0} + J_2 I_{T0} + \rho E_{T0} - \tau R_0) - B_1(P' - B_1 S_0 + \tau R_0)]\frac{t^2}{2} + \cdots \quad (4.31)$$

Similarly, from (2) to (18) of equation (4.1) we obtain the following;

International Research Journal of Applied Sciences, Engineering and Technology
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ISSN (1573-1405);
p –ISSN 0920-5691
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$$S_v(t) = S_{v0} + (\mathrm{r}S_0 - \mathrm{B}_2 S_{v0})t + [\mathrm{r}(\Pi - B_1 S_0 + \tau R_0) - \mathrm{B}_2(\mathrm{r}S_0 - \mathrm{B}_2 S_{v0})]\frac{t^2}{2} + \cdots \quad (4.32)$$

$$S_u(t) = S_{u0} + (K_1 X S_0 - \mathrm{B}_3 S_{u0})t + [K_1 X(\Pi - B_1 S_0 + \tau R_0) - \mathrm{B}_3(K_1 X S_0 - \mathrm{B}_3 S_{u0})]\frac{t^2}{2} + \cdots \quad (4.32)$$

$$S_{vc}(t) = S_{vc0} + (\lambda S_{v0} - \mathrm{B}_4 S_{vc0})t + [\lambda(\mathrm{r}S_0 - \mathrm{B}_2 S_{v0}) - \mathrm{B}_4(\lambda S_{v0} - \mathrm{B}_4 S_{vc0})]\frac{t^2}{2} + \cdots \quad (4.33)$$

$$S_{vn}(t) = S_{vn0} + (K_2 Z S_{v0} - \mathrm{B}_5 S_{vn0})t + [K_2 Z(\mathrm{r}S_0 - \mathrm{B}_2 S_{v0}) - \mathrm{B}_5(K_2 Z S_{v0} - \mathrm{B}_5 S_{vn0})]\frac{t^2}{2} + \cdots \quad (4.34)$$

$$S_{uc}(t) = S_{uc0} + (\sigma S_{u0} - \mathrm{B}_6 S_{uc0})t + [\sigma(K_1 X S_0 - \mathrm{B}_3 S_{u0}) - \mathrm{B}_6(\sigma S_{u0} - \mathrm{B}_6 S_{uc0})]\frac{t^2}{2} + \cdots \quad (4.35)$$

$$S_{un}(t) = S_{un0} + (K_3 Y S_{u0} - \mathrm{B}_7 S_{un0})t + [K_3 Y(K_1 X S_0 - \mathrm{B}_3 S_{u0}) - \mathrm{B}_7(K_3 Y S_{u0} - \mathrm{B}_7 S_{un0})]\frac{t^2}{2} + \cdots \quad (4.36)$$

$$E(t) = E_0 + (\beta_1 S_{vc0} + \beta_2 S_{vn0} + \beta_3 S_{uc0} + \beta_4 S_{un0} - \mathrm{B}_8 E_0)t + [\beta_1(\lambda S_{v0} - \mathrm{B}_4 S_{vc0}) + \beta_2(K_2 Z S_{v0} - \mathrm{B}_5 S_{vn0}) + \beta_3(\sigma S_{u0} - \mathrm{B}_6 S_{uc0}) + \beta_4(K_3 Y S_{u0} - \mathrm{B}_7 S_{un0}) - \mathrm{B}_8(\beta_1 S_{vc0} + \beta_2 S_{vn0} + \beta_3 S_{uc0} + \beta_4 S_{un0} - \mathrm{B}_8 E_0)]\frac{t^2}{2} + \cdots \quad (4.37)$$

$$E_Q(t) = E_{Q0} + (\alpha_1 E_0 - \mathrm{B}_9 E_{Q0})t + [\alpha_1(\beta_1 S_{vc0} + \beta_2 S_{vn0} + \beta_3 S_{uc0} + \beta_4 S_{un0} - \mathrm{B}_8 E_0) - \mathrm{B}_9(\alpha_1 E_0 - \mathrm{B}_9 E_{Q0})]\frac{t^2}{2} + \cdots \quad (4.38)$$

$$E_T(t) = E_{T0} + (\alpha_2 E_0 - \mathrm{B}_{10} E_{T0})t + [\alpha_2(\beta_1 S_{vc0} + \beta_2 S_{vn0} + \beta_3 S_{uc0} + \beta_4 S_{un0} - \mathrm{B}_8 E_0) - \mathrm{B}_{10}(\alpha_2 E_0 - \mathrm{B}_{10} E_{T0})]\frac{t^2}{2} + \cdots \quad (4.39)$$

$$I_T(t) = I_{T0} + (\mathrm{C}_1 E_{Q0} + \mathrm{C}_2 E_{T0} - \mathrm{B}_{11} I_{T0})t + [\mathrm{C}_1(\alpha_1 E_0 - \mathrm{B}_9 E_{Q0}) + \mathrm{C}_2(\alpha_2 E_0 - \mathrm{B}_{10} E_{T0}) - \mathrm{B}_{11}(\mathrm{C}_1 E_{Q0} + \mathrm{C}_2 E_{T0} - \mathrm{B}_{11} I_{T0})]\frac{t^2}{2} + \cdots \quad (4.40)$$

$$I_i(t) = I_{i0} + (\mathrm{C}_3 E_{Q0} + \mathrm{C}_4 E_{T0} - \mathrm{B}_{12} I_{i0})t + [\mathrm{C}_3(\alpha_1 E_0 - \mathrm{B}_9 E_{Q0}) + \mathrm{C}_4(\alpha_2 E_0 - \mathrm{B}_{10} E_{T0}) - \mathrm{B}_{12}(\mathrm{C}_3 E_{Q0} + \mathrm{C}_4 E_{T0} - \mathrm{B}_{12} I_{i0})]\frac{t^2}{2} + \cdots \quad (4.41)$$

$$I_N(t) = I_{N0} + (\mathrm{C}_5 E_{Q0} + \mathrm{C}_6 E_{T0} - \mathrm{d}_3 I_{N0})t + [\mathrm{C}_5(\alpha_1 E_0 - \mathrm{B}_9 E_{Q0}) + \mathrm{C}_6(\alpha_2 E_0 - \mathrm{B}_{10} E_{T0}) - \mathrm{d}_3(\mathrm{C}_5 E_{Q0} + \mathrm{C}_6 E_{T0} - \mathrm{d}_3 I_{N0})]\frac{t^2}{2} + \cdots \quad (4.42)$$

$$R(t) = R_0 + (\mathrm{J}_1 I_{i0} + \mathrm{J}_2 I_{T0} + \rho E_{T0} - \tau R_0)t + [\mathrm{J}_1(\mathrm{C}_3 E_{Q0} + \mathrm{C}_4 E_{T0} - \mathrm{B}_{12} I_{i0}) + \mathrm{J}_2(\mathrm{C}_1 E_{Q0} + \mathrm{C}_2 E_{T0} - \mathrm{B}_{11} I_{T0}) + \rho(\alpha_2 E_0 - \mathrm{B}_{10} E_{T0}) - \tau(\mathrm{J}_1 I_{i0} + \mathrm{J}_2 I_{T0} + \rho E_{T0} - \tau R_0)]\frac{t^2}{2} + \cdots \quad (4.43)$$

$$D_u(t) = D_{u0} + (\mathrm{d}_3 I_{N0} + \mathrm{d}_2 I_{i0} + d_1 I_{T0} - q D_u)t + [\mathrm{d}_3(\mathrm{C}_5 E_{Q0} + \mathrm{C}_6 E_{T0} - \mathrm{d}_3 I_{N0}) + \mathrm{d}_2(\mathrm{C}_3 E_{Q0} + \mathrm{C}_4 E_{T0} - \mathrm{B}_{12} i_{i0}) + d_1(\mathrm{C}_1 E_{Q0} + \mathrm{C}_2 E_{T0} - \mathrm{B}_{11} I_{T0}) - q(\mathrm{d}_3 I_{N0} + \mathrm{d}_2 I_{i0} + d_1 I_{T0} - q D_{u0})]\frac{t^2}{2} + \cdots \quad (4.44)$$

$$S_r(t) = S_{r0} + (\Lambda - \mu_r S_{r0})t + [-\mu_r(\Lambda - \mu_r S_{r0})]\frac{t^2}{2} + \cdots \quad (4.45)$$

$$E_r(t) = E_{r0} + (\omega S_{r0} - \mu_r E_{r0})t + [\omega(\Lambda - \mu_r S_{r0}) - \mu_r(\omega S_{r0} - \mu_r E_{r0})]\frac{t^2}{2} + \cdots \quad (4.46)$$

$$I_r(t) = I_{r0} + (\phi E_{r0} - d_4 I_{r0})t + [\phi(\omega S_{r0} - \mu_r E_{r0}) - d_4(\phi E_{r0} - d_4 I_{r0})]\frac{t^2}{2} + \cdots \quad (4.47)$$

Therefore, equations (4.31) - (4.47) are the analytic solutions of the Ahman *et al.*, (2021) EVD classical model dynamic system of equations with control measures using the Homotopy perturbation method.

Numerical Simulations and Results Discussions

In this section, we shall consider the numerical solution of the large compartmental EVD model.

Table 1: Variable Initial Values and Parameter Values

Variables	Initial Values	Parameter	Values	Parameter	Values
S_0	4396521	P	400	ω	0.8
S_{v0}	1758608	μ	0.00004	τ	0.999
S_{u0}	2637913	μ_r	0.04	ϕ	0.8
S_{vc0}	527582	r	0.4	ρ	0.7862
S_{vn0}	1231026	λ	0.03	α_1	0.95143
S_{uc0}	1582748	σ	0.07	α_2	0.41
S_{un0}	1055165	Λ	2000	θ_1	0.9333
E_0	0	s_1	0.3257	θ_2	0.3014
E_{Q0}	0	s_2	0.55	d_1	0.0288465753
E_{T0}	74	s_3	0.148	d_2	0.00001246575
I_{T0}	500	k_1	0.895	d_3	0.3432465753
I_{i0}	800	k_2	0.04	j_1	0.05186
I_{N0}	400	k_3	0.02	j_2	0.46
R_0	0	β_1	0.04	d_4	0.7110
D_{u0}	24	β_2	0.1		
S_{r0}	6000	β_3	0.5		
E_{r0}	100	β_4	0.3		
I_{r0}	0	q	0 .5		

Using Maple Soft and the values in Table 1we obtained the solutions of equations (4.31) - (4.47) as follows;

International Research Journal of Applied Sciences, Engineering and Technology
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ISSN (1573-1405);
p –ISSN 0920-5691
Impact factor: 6.52

$$\left.\begin{array}{c}
S(t) = 4396521 - 4.119316038\ 10^6 t + 1.930306565\ 10^6 t^2 + \cdots \\
S_v(t) = 1758608 + 1.63754582610^6 t + 8.887274825\ 10^5 t^2 + \cdots \\
S_u(t) = 2637913 + 2.127107168\ 10^6 t + 1.200524546\ 10^6 t^2 + \cdots \\
S_{vc}(t) = 527582 + 31633.85672t + 23929.87758t^2 + \cdots \\
S_{vn}(t) = 1231026 - 54917.85064t + 22195.26974t^2 + \cdots \\
S_{uc}(t) = 1582748 - 6.06783399\ 10^5 t + 2.261567366t^2 + \cdots \\
S_{un}(t) = 1055165 - 2.675265248\ 10^5 t + 59916.42590t^2 + \cdots \\
E(t) = 1.252122938\ 10^6 t + 6.602270550\ 10^5 t^2 + \cdots \\
E_Q(t) = 74 - 69.06716t + 5.974857650\ 10^5 t^2 + \cdots \\
E_T(t) = 2.56686559\ 10^5 t^2 + \cdots \\
I_T(t) = 500 - 226.9290777t + 46.10402364t^2 + \cdots \\
I_i(t) = 800 - 376.9046626t + 80.00712360t^2 + \cdots \\
I_N(t) = 400 - 127.0771285t + 17.03932643t^2 + \cdots \\
R(t) = 649.8800t + 173.5553478t^2 + \cdots \\
D_u(t) = 24 + 139.7318904t + 56.97545005t^2 + \cdots \\
S_r(t) = 6000 + 1760.00t - 35.20000000t^2 + \cdots \\
E_r(t) = 100 + 4796.00t + 608.0800000t^2 + \cdots \\
I_r(t) = 80.0t + 1889.960000t^2 + \cdots
\end{array}\right\} \quad (4.48)$$

We plot solutions of equations (4.48) was obtained using Maple Soft as follows;

International Research Journal of Applied Sciences, Engineering and Technology
Vol.9, No. 06, June-2023;
ISSN (1573-1405);
p –ISSN 0920-5691
Impact factor: 6.52

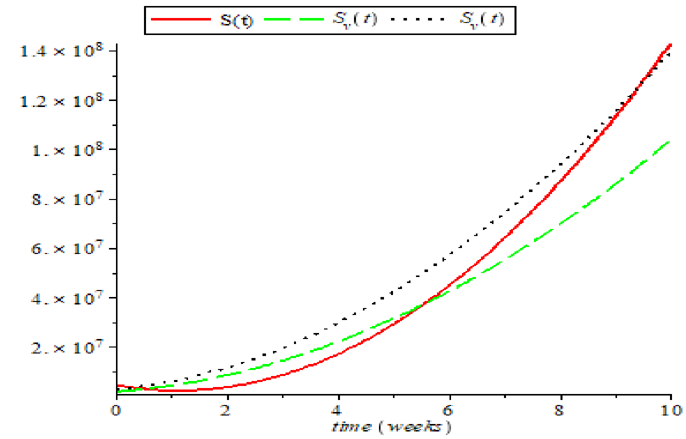

FIGURE 1: GRAPH OF SUSCEPTIBLE POPULATIONS WITH VACCINE

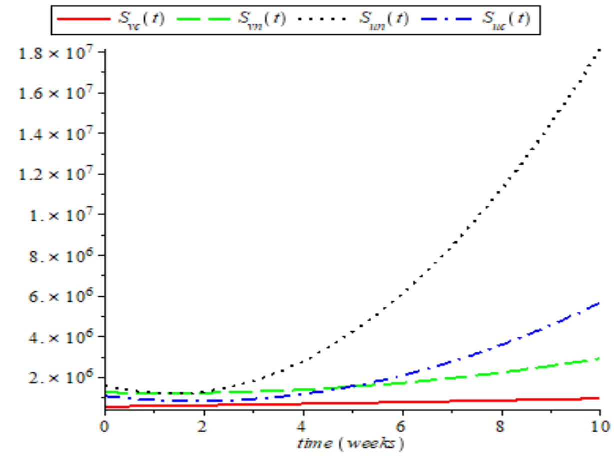

FIGURE 2: GRAPH OF SUSCEPTIBLE POPULATIONS WITH CONDOM

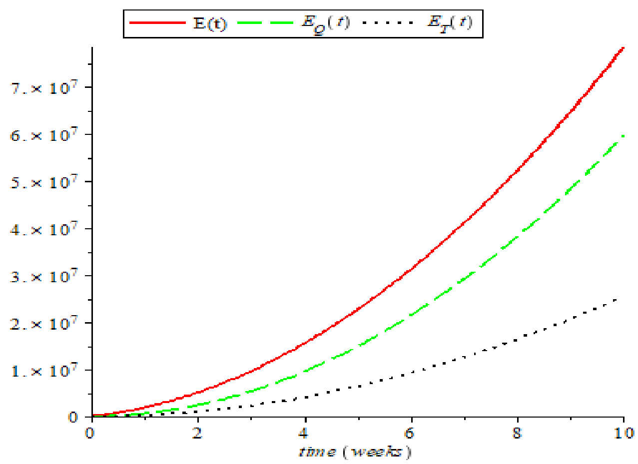

FIGURE 3: GRAPH OF EXPOSED POPULATIONS

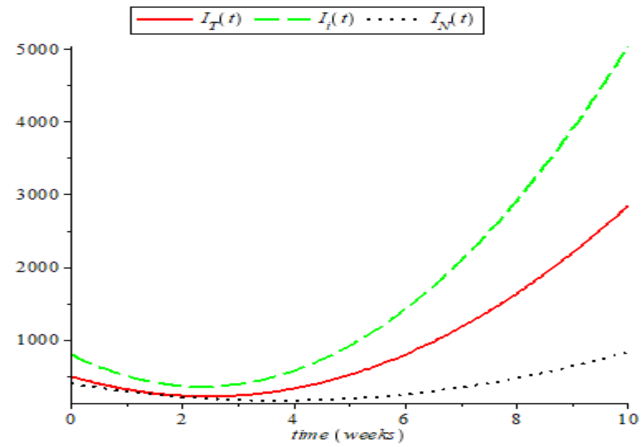

FIGURE 4: GRAPH OF INFECTIOUS POPULATIONS

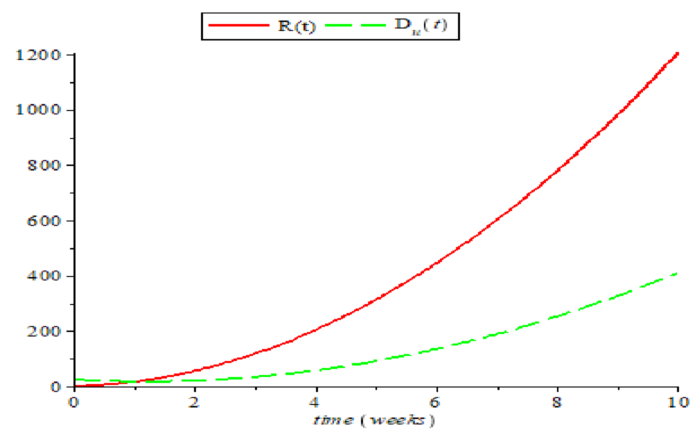

FIGURE 5: GRAPH OF RECOVERED AND DEAD UNBURIED POPULATIONS

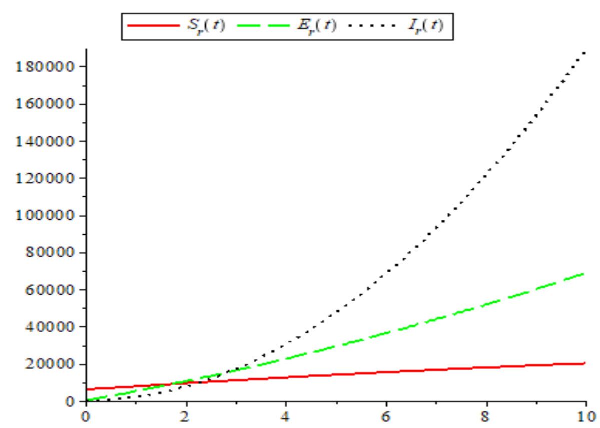

FIGURE 6: GRAPH OF ANIMAL POPULATIONS

Figure 1 is the graph of HPM solution of $S(t), S_v(t)$ and $S_u(t)$, **Figure 2** is the graph of HPM solution of $S_{vc}(t), S_{vn}(t), S_{uc}(t)$ and $S_{un}(t)$,**Figure 3** is the graph of HPM solution of $E(t), E_Q(t)$ and $E_T(t)$, **Figure 4** is the graph of HPM solution of $I_T(t), I_i(t)$ and $I_N(t)$, **Figure 5** is the graph of HPM solution of $R(t)$ and $D_u(t)$ and **Figure 6** is the graph of HPM solution of $S_r(t), E_r(t)$ and $I_r(t)$.

Figures 1- 6 shows that for each compartment of the model the HPM solution is approximate to the exact solution of the EVD model.

4. Conclusion

In this work, we considered a large eighteen compartmental EVD epidemic model with control measures which is an extension of Ahman et al., (2021); we applied the Homotopy Perturbation Method (HPM) in solving the large EVD model and obtained the solution of zeroth, first and second order. The simulation of the HPM solution of each compartment gives a very good result. We have been able to show that HPM can solve large compartmental disease model. We therefore conclude that HPM is valid for solving a large number of parameters and equations models.

Competing Interests

There are no competing interests of any kind among the authors.

Acknowledgment

No Acknowledgments.

Availability of Data

All data are available in the work.

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