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TEACHING MATHEMATICS FOR CRITICAL THINKING, ESSENTIAL SKILL FOR EFFECTIVE LIVING

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ABSTRACT

One of the important aims of mathematics instruction is to develop Critical Thinking in the learner which will enable him to live effectively in the Society. However, it has been reported that students graduate in Mathematics, from school, without training in critical thinking. A major Factor for this situation is that teachers lack knowledge of how to teach Mathematics for critical thinking. This paper tried to provide some strategies for teaching critical thinking in mathematics instruction.

INTRODUCTION

Mathematics is one of the few subjects, which are, in most countries, taught to all students every year, throughout many years of schooling. Why does society give such attention and lots of time to the teaching and learning of mathematics?

Some reasons have been proffered for this phenomenon, which include that: Mathematics is an essential part of human culture, which the educational system is designed to transmit; mathematics is the best way to teach youngsters how to think. Pollak (1986:46), believed that the "most fundamental reason why we place so much emphasis on mathematics is its usefulness." Mathematics is integral to everything about life. Every occupation, which our students may choose to pursue, and much of their everyday lives, are full of the opportunity and the need to apply mathematics. But, despite this usefulness of mathematics, and the emphasis placed on its teaching and learning, a major problem exists. Many children seem to have major mental barriers to mathematics (Arnold, 1986). An increasing number of students seem unable to succeed in mathematics (WAEC, 1986: Lesh, 2003). Mathematics anxiety is widespread and too many students avoid enrolling in mathematics courses unless they are strictly required. Although many factors, including social, psychological, societal factors are certainly at the root of this situation, the role played by teacher's method could also be most crucial.

For example, the following episode drawn from classroom experience

Demonstrates:

Teacher: Good morning children! Today, we have "Division by a fraction"
What did I say?
Class: Division by a fraction.
Teacher: To divide by a fraction, simply invert the divisor and multiply.
Now, let's solve some examples.
Example: $2/3 \div ¾$
The divisor is ¾, and its inverse is 4/3, so
$2/3 \div ¾ = 2/3 \times 4/3 = 8/9$.
Pupil: Teacher, why must we invert the divisor and multiply? And teacher, if we invert the first fraction, shall we still arrive at the same result?
Teacher: Please, sit down, and don't waste our time.
(Source: Personal Encounter in a classroom).

It is clear from this episode, that the teacher simply wanted the pupils to Learn and memorize the rule for division by a fraction. Memorizing prevents children from actively thinking about and figuring out solutions for themselves. This makes them to have little, if any, mental growth (Wakefield, 1997). Secondly, the teaching of mathematics in this case, is authoritarian. The teacher acted as the fountain of wisdom, and the boss. Children were denied opportunity to ask questions and err their own views. Thirdly, it is not surprising that the child in this episode would never raise his voice again in this teacher's class, having been hushed down and disgraced by the teacher. The child would never learn how to think, and would remain stunted in learning.

Thus, a major problem to the mental barriers which children have in Mathematics could be the lack of training in critical thinking in mathematics instruction. The Foundation for critical thinking (F.C.T)(2004) reported that students reach high school, college and b beyond without Training in critical thinking. The situation cannot be any better in Nigeria. F.C.T. noted that without critical thinking systematically designed into Instruction, learning is typically transitory and superficial.

The question is, are our school teachers knowledgeable in teaching mathematics for critical thinking? How can critical thinking be taught in mathematics instruction? Although some researchers agree that critical thinking should be taught in mathematics instruction, they do not explain what a teacher should do or how a teacher should teach mathematics for critical thinking. This is the problem that this paper seeks to address.

The paper is organized as follows:

- Meaning of Mathematics.
- Meaning of Critical Thinking.
- Role of mathematics in Developing Critical Thinking.
- Mathematics Concepts and techniques that Aid Development in Critical Thinking.
- Strategies for teaching mathematics for critical thinking.

MEANING OF MATHEMATICS

The definitions of mathematics are time dependent. It depends on what hour of the day and whom you ask (Davis, 1986). For instance, for Davis, mathematics is the science of quantity and space; or it is the science of deductive structures; or it is the science of abstractable patterns.

However, in a study on students' views of mathematics, the students Perceived that mathematics consists of a predetermined set of rules and Procedures "passed on' by teachers to the next generation (Borasi, 1990). This assumption, of course would not allow students to consider thinking on their own as an appropriate strategy to approach mathematical problems. Other beliefs about mathematics commonly held by students are; Mathematics is passed on 'from above', by experts, so one must take a Passive role on learning it; formal mathematics has little to do with thinking or discovery (Burkhardt, et al 1986).

Lappan and Schram, (1998), defined mathematics as a way of thinking about and organizing ones experiences. Mathematics involves thinking, conjecturing, modeling, and describing all aspects of reasoning about situations.

MEANING OF CRITICAL THINKING

Thinking is defined as "information processing (Kilpatrick, 1986). Critical thinking therefore implies very important information processing upon which future knowledge depends.

According to Pollak (1986), critical thinking involves "mental wrestling". This implies that critical thinking is a conscious process (Zant, 1952). It involves careful steps such as definite statement or at least a careful consideration of the assumptions, a knowledge of the meaning and procedures of inductive reasoning, the ability to make logical inferences, the ability to recognize logical fallacies both in one's own work and in the work of others, the ability to know that the proof or solution of a problem is complete, and finally that the correctness of any solution is dependent on the assumptions and the validity of solutions of dependent problems or theorems.

The foundation for critical thinking (F.C.T.) (2004:1), defined critical Thinking as "the intellectually disciplined process of actively and skillfully Conceptualizing, applying, analyzing, synthesizing, and/ or evaluating Information gathered from, or generated by observation, experience, Reflection, reasoning, or communication, as a guide to belief and action". It entails the examination of those structures or elements of thought implicit In all reasoning: purpose, problem, or question-at-issue; assumption; Concepts; empirical grounding; reasoning leading to conclusions; Implications and consequences, objections from alternative viewpoints; and frame of reference.

These definitions suggest that critical thinking is not a single skill, but a battery of skills. The critical thinking skills are: ability to distinguish between facts and opinions; draw valid conclusions from given data; identify assumptions underlying conclusions; identify the limitations of given data; ability to speculate, imagine and see connections among ideas.

According to F.C.T. (2004:2), a well cultivated critical thinker:

(i). Raises vital questions and problems formulating them clearly and precisely;

(ii). Gathers and assess relevant information, using abstract ideas to interpret it effectively, comes to well-reasoned conclusions and solutions, testing them against relevant criteria and standards;

(iii) Thinks open mindedly within alternative systems of thought, recognizing and assessing, as need be, their assumptions, implications, and practical consequences; and

(iv). Communicates effectively with others in figuring out solutions to complex problems.

From the foregoing, it is clear that critical thinking is self-directed, self-disciplined, self-monitored, and self-corrective. Also, items (i)-(iii) above refer to problem solving abilities. Hence, critical thinking can be said to entail effective communication and problem solving abilities. Problem solving abilities are mathematical abilities.

THE ROLE OF MATHEMATICS IN DEVELOPING CRITICAL THINKING

One of the most important aims considered in education is to help children Have the appetite and ability to think analytically and critically, to be able to Speculate and imagine, to see connections among ideas, and to be able to use What they know to enhance their own lives and to contribute to their culture (Eisner, 1997).

Thus, the development of critical thinking is an important aim of education, which must be taught to students. According to F.C.T. (2004), inability to develop critical thinking makes most of us less than what we could be. It makes most of our great capacity dormant and most undeveloped. One of the core subjects in the school curricular, for achieving development in critical thinking is mathematics. This was suggested by Zant (1952), when he pointed out that the

ability to critical thinking in life situations may be improved by giving attentions to the process in ideal (mathematical) situations. He said further that, mathematical thinking may be thought of as furnishing the ideal for all other critical thought. This is supported by Cockcroft's (1986) report. It is also supported by Pollak (1986), who stated that mathematics is the best way to teach youngsters how to think. According to him, we teach mathematics for its impartation of reasoning power.

Reasoning is a central part of mathematical activity (N.C.T.M., 2000). It is a defining feature of mathematics. The process of thinking and reasoning are engaged in the process of mathematical problem solving. Thus, training in the processes of mathematical problem solving provides training in reasoning and critical thinking.

Much of mathematics involves analytical reasoning. Infact, mathematical problem solving is often a matter of reasoning analytically, constructing an image, using the image to support additional conceptual reasoning, which in turn may suggest an elaborate image (Wheatley and Cobb, 1990). People who reason and think analytically tend to note patterns, structure, or regularities in both real-world situations and symbolic objects; they ask if those patterns are accidental or if they occur for a reason; and they conjecture and prove. Conjecture, which means informed guessing, is a major pathway to discovery and doing mathematics involves discovery. Thus, effective training in mathematics involves the learner in discovery whose pathway is conjecturing which is critical thinking skill.

Mathematics is the activity of creating relationships, many of which are based in visual imagery (Wheatley and Cobb, 1990). It involves examining patterns, and noting regularities, making conjectures about possible generalization and evaluating the conjectures, and using inductive and deductive reasoning to formulate mathematics arguments. Thus, mathematics involves all the critical thinking abilities, which include analytical reasoning, reflective thinking, inductive reasoning and deductive reasoning. Thus, the learning of mathematics should develop critical thinking in the learner. Hence, critical thinking strategies should be taught in mathematics instruction. Skemp (1976), pointed out that one reason for teaching thinking strategies is to help children view mathematics as an activity that is supposed to make sense rather than one that involves memorized rules. Some mathematics concepts and some teaching techniques have been found to provide easy vehicle for teaching critical thinking.

MATHEMATICS CONCEPTS AND TECHNIQUES THAT AID DEVELOPMENT IN CRITICAL THINKING

Although reasoning and thinking are not special activities reserved for special times or special topics in the curriculum, but there are some mathematics topics/ concepts and some techniques that effectively and easily aid development in critical thinking. These are as follows:

(1) Geometric Proofs

Proof is conceived of as validation (Davis, 1986). Validation of a statement is the process of proving that the statement is true or correct. Davis went further to give several purposes, which proof serves, such as; proof is discovery because proof can lead to discovery of new mathematics; proof involves formalization; proof is a debating forum... proof is an argument. Proof passes from assumption to conclusion by tiny logical chains of reasoning.

According to N.C.T.M. (2000), a mathematical proof is a formal way of expressing particular kinds of reasoning and justification. This idea was earlier expressed by Zant (1952), when he stated that the fundamental concepts of mathematical thought as exhibited clearly in

elementary Euclidean geometry furnish as excellent example of critical thinking. The proof of a theorem involves undefined terms, definitions, assumptions, and deductive reasoning. Infact, deductive reasoning is the logical foundation of formal proofs of theorems. As stated earlier on, critical thinking also involves careful consideration of assumptions, procedures of inductive reasoning and the ability to make logical inferences based on deductive reasoning. Hence, proofs of geometric theorems provide opportunity for students to learn critical thinking.

(2) Logic

Logic involves logical reasoning. And logical reasoning is deductive reasoning (Sera, 1993). According to Sera, deductive reasoning (logical reasoning) is the process of demonstrating that if certain statements are accepted as true, then other statements can be shown to follow from them. Mathematics itself is a deductive science, and deductive reasoning is involved both in logic and critical thinking. Hence, the study of logic is a sure vehicle for development of critical thinking.

(3) Word Problems

Another vehicle for teaching children to think is through mathematical word problem solving. The processes of thinking are engaged in the processes of mathematical word problem solving. These include reading and understanding the problem, analyzing the problem, identifying relationships between the given facts, determining what it is that is being asked for, organizing the information in the problem using diagrams/tables, using just the needed information, solving mini problems, translating the problem, etc.(Burkhardt et al, 1986). These involve reasoning. Bebout and Carpenter (1989), pointed out that the use of realistic word problems can help children continue to build the mathematical thinking strategies. Mathematical thinking strategies relate to critical thinking strategies.

(4) Other mathematics concepts that have advantage of being the source of particularly appropriate and natural experiences for developing communication and reasoning skills are: estimation, measurement, statistics, probability, number patterns etc.

(5) Mathematical Games.

A learning environment that provides the opportunity for children to interact and thus to exchange views fosters and supports focused thinking in children (Wakefield, 1997). Game setting provides such opportunities. The debate and exchange of views that occur quite naturally during dice, card, and board games encourage children to examine their own thinking.

The normal interactions of children at play (e.g. figuring out how many spaces to move a game piece according to the roll of the dice, what combination nof cards adds up to 15, or what a chance card requires) provide natural challenges that involve reasoning and thinking.

Games provide children opportunity to be successful and have a feeling of achievement. When children are successful, they feel confident about what they can do. They see themselves as being "smart" and self-esteem rises (Wakefield, 1997). This sense of success and competence helps create an optimal environment for making more challenging thinking.

According to Ainley, as cited in Agwagah (2001), the mathematical thinking processes involved in games are: predicting and testing, conjecturing, generalizing, and checking and justifying. These are critical thinking processes. Hence, to develop critical thinking in children,

mathematical games should be used in mathematics instruction. However, most teachers feel that playing games have no place in mathematics instruction (Agwagah, 2001).

STRATEGIES FOR TEACHING MATHEMATICS FOR CRITICAL THINKING

(1) Many teachers have tried to and still teach mathematics with the traditional drill-and –practice approach that emphasize memorization of facts and procedural practice on computation (Ezema, 2000). This traditional approach should be replaced with approach that encourages children to think and figure things out themselves. It is no longer sufficient for students, in this millennium, to enter the working world with only disconnected rules, theorems, and techniques stored in their mathematics heads, What is valued in business and industry is being able to think and reason mathematically and to bring the powers of mathematics to bear on a problem that needs a solution (Lappan and Schram, 1989). Making sense of things, perceiving structures, seeing relationships, and analyzing them in order to explain why something is as it seems is what mathematical thinking and reasoning mean.

(2) In teaching mathematics for critical thinking, the teacher should conduct lessons that help students achieve critical thinking-level objectives. The strategy is to conduct lessons so that students feel free to ask questions, make mistakes, and disagree with ideas, even the teacher's ideas.

(3) Students should be exposed to heuristic activities such as brainstorming, open-ended question sessions, and discussions in which ideas for consideration are examined regarding purpose, structure, advantages, and disadvantages.

(4) Teachers should give students opportunities to explore mathematical situations with a real sense of inquiry – asking questions, talking about ideas, looking for patterns, and reasoning about what makes sense.

(5) Teachers should give students opportunities to make and investigate conjectures. Teachers can achieve this by asking questions such as: What do you think will happen next? What is the pattern? Is this true always? Sometimes? By making and investigating conjectures, students learn to answer the question, why does this work? The manner of posing questions by teachers may also help students learn to conjecture. For instance, instead of saying, "show that the mean of a set of data doubles when all the values in the data set are doubled", a teacher might ask," suppose all the values in the data are doubled, what change, if any, is there in the mean of the sample?" Why?

(6) In any mathematics lesson, teachers should help students understand that assertions should always have reasons. Questions such as, "why do you think it is true?", and "does any one think the answer is different?", and "why do you think so?", help students see that statements need to be supported or refuted by evidence. Students need to learn and agree on what is acceptable as an adequate argument in the mathematics classroom. These are the first steps toward realizing that mathematical reasoning is based on specific assumptions and rules.

(7) Teachers should give students opportunities to learn reasoning through class discussion of claims that other students make. For instance, the statement, if a number is divisible by 6 and by 4, then it is divisible by 24, could be examined in various ways. Some students could find a counter example – the number 12 is

divisible by 6 and 4, but not by 24. Thus, both the plausible and flawed arguments offered by students could create opportunity for discussion.

(8) Teachers should give students opportunity to assume responsibility for as much of the mathematical activities as possible. For instance, in engaging students in a mathematical game, the teacher should allow them to "invent", for instance, their own ways of keeping score, or their own rules.

CONCLUSION

It is clear that mathematics plays an important role in the development of critical thinking. However, it has been reported that training in critical thinking is never part of mathematics instruction. The major factor being that teachers lack the strategies for teaching critical thinking in mathematics instruction. This paper has tried to solve this problem by providing some strategies.

It is recommended, therefore, that teachers should regularly engage students in thinking and reasoning in the classroom. The responsibility of the mathematics teacher ends when he has taught his students the fundamentals of critical thinking and when he has taught them to apply these skills in a wide variety if instances in the mathematical field.

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