ROBABILITY THEORIES

AND

DISTRIBUTIONS 1

Deterministic Models are Models whose values can be determined without error 1) = IR, U=u+ at, y= 2x + x2 Deterministic Models are predictable

PROBABILITY THEORIES AND DISTRIBUTIONS 1

(OTOMEOHA SERIES)

DR. NDIDIAMAKA OZOFOR

PROBABILITY THEORIES

AND

DISTRIBUTIONS 1

(OTOMEOHA SERIES)

NDIDIAMAKA MIKE OZOFOR

(B.Sc., Med., Ph.D.)

Published 2001Ø by
GOD'S WILL PRINTS ENTER.
#12 Udoji Street Ogui New Layout, Enugu.
Phone: 08065818580, 08073505717.

ISBN: 978-2473-71-5

The Mean and Variance of the Poisson Dsn

$X:\underline{P}(\lambda)$

Then $\underline{P}\,[X = r] = \frac{\lambda^r e^{-\lambda}}{rc!}$ $\quad r = 0,1,2\ldots$

$Ex = \sum_r \underline{P}[x = r] = \sum r \frac{\lambda^r e^{-\lambda}}{r!}$

$= \lambda \quad \sum \left[\frac{\lambda^{r-1} e^{-\lambda}}{(r-1)!} \right]$

$Ex = \lambda \sum \frac{\lambda^{r-1} e^{-\lambda}}{(r1)!}$ | $\frac{\sum \lambda^r e^{-\lambda}}{r!}$

$Ex = \lambda$

Show that Var $X = \lambda$

The Geometric Dsn

$X:G(P)$

$\underline{P}[X=r] = (1p)p^r$ $\quad r = 0, 1,2$

$EX = \sum_r (1P)P^r = (1-P)\sum rP^r$

$= (1-P)\ [1.P + 2P^2 + 3P^3 + 4P^4\ldots]$

$$
\begin{aligned}
= (1-P)\ [P + P^2 + P^3 + P^4 + P^5 \ldots \\
P^2 + P^3 + P^4 + P^5 \ldots \\
P^3 + P^4 + P^5 \ldots \\
P^4 + P^5 \ldots
\end{aligned}
$$

$P^5 + P^6 \ldots$

$(1\text{-}P) \left(\frac{P}{(1\text{-}P)} + \frac{P^2}{(1\text{-}P)} + \frac{P^3}{(1\text{-}P)}\ l\right)$

$(P + P^2 + P^3 + \ldots) = \frac{P}{(1\text{-}P)} = \underline{\underline{P(1\text{-}P)^{-1}}}$

$Var(X) = EX^2 - (EX)^2$

$EX^2 = \sum r^2 \underline{P}[X = r]$

$= \sum(r(r\text{-}1) + r) P [X = r]$

$= \sum r(r\text{-}1) \underline{P}[x = r] + \sum r P[x=r]$

$= \sum r(r\text{-}1)(1\text{-}P) P^r +$

$= (1\text{-}P) \sum r (r\text{-}1) P^r +$

$= (1\text{-}P) P^2 \sum \frac{d^2}{dP^2} \sum P^r +$

$= (1\text{-}P) P^2 \sum \frac{d^2}{dP^2} \frac{1}{(1\text{-}P)} +$

$= 2(1\text{-}P)P^2 (1\text{-}P)^{-3} + P(1\text{-}P)^{-1}$

$= \frac{2P^2}{(1\text{-}P)^2} + \frac{P}{(1\text{-}P)}$

$Var\ X = \frac{2P^2}{(1\text{-}P)^2} + \frac{P}{(1\text{-}P)} = \frac{P^2}{(1\text{-}P)^2}$

$= \frac{2P^2}{(1\text{-}P)^2} + \frac{P}{(1\text{-}P)} = \frac{P^2 \times P(1\text{-}P)}{(1\text{-}P)^2} = \frac{P}{(1\text{-}P)^2}$

$Ex = (1\text{-}P) \sum^r P^r$

$= (1\text{-}P) \sum \frac{d}{dp} P^r .P$

$= (1\text{-}P)P \sum \frac{d}{dp} P^r$

$= (1\text{-}P)P \frac{d}{dp} \sum P^r$

$= (1\text{-}P) P \frac{d}{dp} \left(\frac{1}{12}\right)$

$\underline{\sum P^r\ P + P^2 + P^3 + \ldots}$

$Ex = (1\text{-}P)P \frac{1}{(1\text{-}P)^2}$

$= \frac{P}{(1\text{-}P)} = P(1\text{-}P)$

$y = P^r$

$\frac{dy}{dp} = rPr\text{-}r$

$\frac{d^2y}{dpp^2} = r(r\text{-}1)\ Pr\text{-}2$

$y = (1\text{-}P)^{-1}$

At Home

1. $X:f(x) = \frac{1}{2x^2}$ $\quad x \geq 1$

$\frac{1}{2}$ $\quad o \leq x \geq 1$

calculate EX

2. X: b(n:p)

$\underline{P}[X = n] = 0$

$\underline{P}[X = r] = K\, {}^nC_r P^r q^{n-r}$

i. find K

ii. EX

The Uniform Dsn

U(a,b)

$$(x) = \frac{1}{b-a}$$

$$X = \int_a^b x\, f(x)\, dx$$

$$= \int \frac{x}{b-a}\, dx = \left.\frac{x^3}{2(b-a)}\right]_a^b$$

$$= \frac{b^2 - a^2}{2(b-a)} = \frac{b+a}{2}$$

$$EX^2 = \int_a^b x^2 f(x) dx$$

$$= \int_a^b \frac{x^2}{b-a}\, dx = \left.\frac{x^3}{3(b-a)}\right]_a^b$$

$$EX^2 = \frac{b^3 - a^3}{3(b-a)}$$

$$\left(\frac{b+a}{2}\right)$$

$$\text{Var } X = EX^2 - (EX)^2$$

$$\frac{b^2 + 2ab + a^2}{4}$$

$$= \frac{b^3 - a^3}{3(b-a)} - \frac{(b+a)^2}{4}$$

$$(b - a)(b^2 - 2ba + a^2)$$

$$= \frac{(b - a)(b^2 + ba + a^2)}{3(a - b)} - \frac{(b^2 + 2^1 ba + a^2)}{4}$$

$$= \frac{b^2 - ba + a^2}{3} - \frac{(b^2 - 2ba + a^2)}{4}$$

$$\frac{4(b^2 - ba + a^2) - 3(b^2 - 2ba + a^2)}{12}$$

$$= \frac{b^2 - 2ab + a^2}{12} = \frac{(b-a)^2}{12}$$

The Exponential Dsn

X: E(λ), $f(x) = \lambda e^{-\lambda x} \quad x \geq 0$

Recall the Pdf of the gamma ds if X has gamma (λ r)

$$F(x) = \frac{\lambda(\lambda x)^{r-1} e^{-\lambda x}}{(r-1)!} \qquad x \geq 0$$

We obtain EX and Var X when X:E(λ)

Noted
The sum of
rth Exponen
To Gamma

$$EX = \int_0^\infty x f(x)dx = \int_0^\infty x\lambda e^{-\lambda x}\, dx$$

$$= \frac{1}{\lambda}\int_0^\infty (\lambda x)e^{-\lambda x}dx = \frac{1}{\lambda}$$

= gamma Dsn with r = 2

$$EX^2 = \int_0^\infty x^2 f(x)dx = \int_0^\infty x^2\lambda e^{-\lambda x}\, dx$$

$$= \frac{1}{\lambda^2}\int_0^\infty \frac{2\lambda(\lambda x)^2 e^{-\lambda x}}{2!}dx = \frac{1^{\wedge 2}}{\lambda^2}\int \frac{\lambda(\lambda x)^2 e^{-\lambda x}}{2}$$

for r = 3
= gamma Dsn
$= \frac{\lambda(\lambda x)^2 e^{-\lambda x}}{2!}$

$$EX^2 = \frac{2}{\lambda^2}$$

$$\text{Var } X = EX^2 \quad (EX)^2$$

$$= \frac{2}{\lambda^2} - \left(\frac{1}{\lambda}\right)^2 = \frac{2}{\lambda^2} - \frac{1}{\lambda^2}$$

$$\text{Var } X = \frac{1}{\lambda^2}$$

If X: E(0.4)

$$EX = \frac{1}{0.4} = 2.5$$

$$EX^2 = \frac{2}{\lambda^2} = \frac{2}{.16} = 12.5$$

$$\text{Var } aX = 12.5 - 6.25 = 6.25$$

$$= a^2\text{Var} = 4 \times 6.25 = 25$$

Gamma Dsn

Find EX, Var X

$$EX = \int_0^\infty x f(x)dx = \int_0^\infty \frac{x\lambda(\lambda x)^{r-1} e^{-\lambda r}}{(r-1)!} dx$$

$$= \int_0^\infty \frac{(\lambda x)^r e^{-\lambda x}}{(r-1)!}$$

$$= \int_0^\infty \frac{\lambda r}{\lambda} \frac{(\lambda x)^r}{r!} e^{-\lambda x} dx \qquad (r-1)! = \frac{r!}{r}$$

$$= \frac{r}{\lambda}\underbrace{\int \lambda \frac{(\lambda x)^r e^{-\lambda x}}{r!} dx} = \frac{r}{\lambda}$$

$$EX = \frac{r}{\lambda}$$

$$EX^2 = \int_0^{\infty} x^2 f(x)dx$$

$$= \int_0^{\infty} \frac{\lambda(x\lambda)^{r!} e^{-\lambda x}}{(r-1)!} dx$$

$$Z = X_1 + X_2 + \ldots X_r \text{ has aga}$$

$$EZ = E(X_1 + X_2 + \ldots X_4)$$
$$= E X_1 + EX_2 + \ldots E X_n$$
$$= \frac{1}{\lambda} + \frac{1}{\lambda} + \ldots \frac{1}{\lambda}$$
$$= \frac{r}{\lambda}$$

$$\int_0^{\infty} x \frac{(x_1)(\lambda x)^{r-1} e^{-\lambda x}}{(r-1)!} dx$$

$$\int_0^{\infty} x \frac{(\lambda x)^r}{(r-1)!} e^{-\lambda x} =$$

$$= \int_0^{\infty} \frac{a^2 \lambda(\lambda x)^{r-1} e^{-\lambda x}}{(r-1)!} dx = \frac{r(r+1)}{\lambda^2} \int_0^{\infty} \frac{\lambda (\lambda x)^{r+1} e^{-\lambda x}}{(r+1)!}$$

$$= \frac{r(r+1)}{\lambda^2}$$

$$\text{Var } X = EX^2 - (EX)^2$$
$$= \frac{r(r+1)}{\lambda^2} - \frac{r^2}{\lambda^2} = \frac{r^2 + r - r^2}{\lambda^2} = \frac{r}{\lambda^2}$$

$$\text{Var } X = \frac{r}{\lambda^2}$$

blem

The weight of rods made by a company has the normal dsn h ,eam 10kg and s.d 4kg. A length of rod is sold for ₦10 it weighs re than 16kg. it is sold for ₦8.50 if it weighs less than 6kg. erwise it is sold for ₦9.00. Find mean of the selling a price of gth of rod picked at random.

A customer picked 100 of such rods at random. How many of re are expected to weigh more than 16kg, less than 6kg and ween 6kg and 16kg.

xpl

A gambler wins ₦1.00 if 1 appears in a throw of a die, ₦2.00 is throw of a die, ₦2.00 if 2013 appears in ₦0.50. Otherwise if X is his winging. Find EX, Var X.

2. The r.v. has Thed dsn

$\underline{P}$ [X = n] = 0

$\underline{P}$ [X = r] = $K\, nC^r P^r q^{n-r}$ r = 0.1 … n-1

Obtain EX

Solu 1: EX = ∑k P[X = k)

₦1 x P [X = 1] + ₦2 x P (X = 2] + 0.50 x P [X= 3]

= 1 x P [oe appears] + 2 x P [20 or 3 appears]

= + 0.5 $\underline{P}$ [otherwise]

$= 1 \times {}^{1}/_{6} + 2 \times {}^{2}/_{6} + 0.5 \times {}^{3}/_{6}$

$= 1 \times {}^{1}/_{6} + 2 \times ⅓ + + 0.5 \times ½ =$

$EX^2 = \sum k^2 P[X = k]$

$= 1^2 \times P[X=1] + 2^2 P[x = 2] + 0.5^2 P[X = 0.5]$

Var X =

Chybeysher's Inequality OR Upper bond of Probabil

X is a r.v and C is any constant suppose

$E(X<)^2 < \infty$ and ε is any non-negative number i.e. $\varepsilon >$

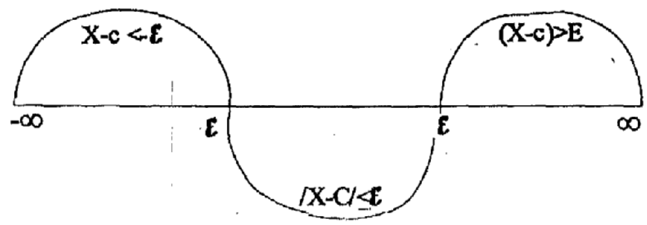
Then

$$\underline{P} \quad X - < 1 \geq E] \geq \frac{E(X-<)^2}{E^2}$$

Proof: $$E(X - <)^2 = \int_{-\infty}^{\infty} (x-<)^2 \; f(x)dx$$

For x confi

1 X < 1> E



$E(X<)^2$

$$= \int_{1x \; < 1 \geq \varepsilon} (x-<)^2 \; f(x)dx \quad + \quad \int_{1x-<1 \;\; \leq \varepsilon} (x-<)^2 \; f(x) <x$$

$$\geq \int_{1 \; x \; -<1 \geq 2} \varepsilon^2 \; f(x)dx \; = \Sigma^2 \int_{\lambda -</ \geq 2} ff(x)dx$$

$$= \varepsilon^2 \underline{P} \; [1 \; X - < 1 \geq \varepsilon] \; \leq E \; (X<)^2$$

$$\underline{P} \; [1 \; X - < 1 \geq \varepsilon] \; \leq \frac{E \; (X<)^2}{E^2}$$

When $c = \mu = EX$ we have

$$P \; [1X \;\; \mu 1 \geq \mu \geq \varepsilon] \; \leq \; \frac{S^2}{\varepsilon^2}$$

ample: Obtaining the chebysher's upper bound and compare witl exact of the ff if X : N(9,2)

(i) P [X-9] ≥ 3]

(ii) P [X-7]≥5]

lu: Chebysher's Upper bound

$$[X-9]\geq 3] \leq \frac{E(X-9)^2}{3^2} = \frac{4}{9}$$

act Probability

$$P\,[X\text{-}9]\geq 3\ ]\geq 3\,] = P\left(\frac{|x-9|}{2} \geq \frac{3}{2}\right)$$

= P [|Z|≥1.5]

= 2 x area from 1.5 to ∞

= 2 x 0.0668

= 0.1336

Chybysher's Inequality

If X ∪ (0,5), obtain the chebysher's upper bounds and co with the probabilities of the ff.

(i) P [1X-2 1≥]

(ii) P [1X-2.5 1 ≥2]

For the same X above, plot the graph of the upper bound ff P [1X – 3[≥k] for different values of k. on the same graph pl exact probabilities.

Soln:

(i) P [1X – 2 1≥ 1] ≥ $\frac{E(X-2)^2}{1}$

$F(x-2)^2 = EX^2 - 4EX + 4$

$EX^2 = \int \frac{x^2}{5} dx = \frac{x^3}{15}\Big]$

$= \frac{125}{15} = \frac{25}{3}$

$EX = \int xf(x)\, dx = \frac{5+0}{2} = 2.5$

$E(X-2)^2 = \frac{25}{3} \quad 4 \times \frac{5}{2} + 4 = \frac{7}{3}$

P [1X2/≥/] ≤ 2.33

(ii) P[1X-2.5/≥2] ≤ $\frac{E(X-2)^2}{2^2}$ =

$\frac{\underline{E(X2.5)^2}}{4} = \text{Var } X = \frac{25}{3} - \frac{25}{4} = \frac{25}{12}$

$= \frac{25}{3} \times \frac{1}{4} = \frac{25}{48} = 0.52$

Exact Probability

$\underline{P}\,[1X\text{-}2.5/\geq 2] = \underline{P}\,[X\ \ 2.5 \geq 2 \text{ or } X\ 2.5 \leq -2]$

$= \underline{P}\,[X\geq 4.5 \text{ or} \leq 0.5\,]$

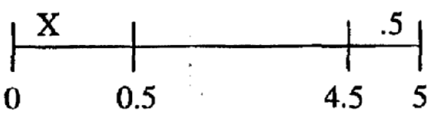
$= \underline{P}\,[X\geq 4.5 \text{ or } X\leq 0.5]$

$= \frac{1}{10} + \frac{1}{10} = 0.5$

$1 - P\,[\,X < 4.5]$

$1 \quad \frac{.5}{5}$

$= \frac{5}{10} \quad \frac{1}{10}$

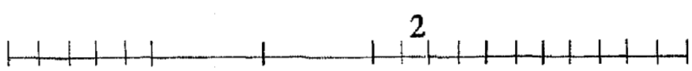


$\frac{0.5}{5}$ $\qquad$ $2(X+2.5)$

$P[1X - 2\,/\geq/]$ $\qquad$ 12.5

$= \underline{P}\,[X - 2 \geq 1 \quad \text{or} \quad X - 2 \geq -1\,]$

$= \underline{P}\,[X\geq 3] + \underline{P}\,[X\ 1]$



$= \frac{2}{5} + \frac{1}{5} = \frac{3}{5} = .6$

The Poisson Dsn

The random variable X is said to have the Poisson dsn parameter λ if its probability function is

$$\underline{P}[X = r] = \frac{\lambda^r e^{-\lambda}}{r!} \qquad r = ,1,2\ldots$$

The Poisson dsn is an important dsn used to model physical systems.

Example: If X has the $\underline{P}(0.3)$, x calculate the ff probability
(i) P [X = 2] (ii) P[X≥2] (iii) P [2<x<5]

(i) $$P[X = r] = \frac{0.3^r e^{-0.3}}{2!} = \frac{0.2e^{-0.3}}{2} = 0.33$$

(ii) $$P[X \geq 2] = 1 - P[X<2]$$

$$= P[X = 0] + P[X = 1]$$

$$= \frac{(0.3)^0 e^{-0.3}}{0!} + \frac{(0.3)^1 e^{-0.3}}{1!}$$

$$= 1 \quad \frac{-(0.3)^0 e^{-0.3}}{0!} - \frac{(0.3)^1 e^{-0.3}}{1!}$$

1 - 0.7408 - 0.2224567
= 1 - 96504
= 0.0369

2. X has a Poisson dsn. If
$\underline{P}[X = 2] = 0.8\ \underline{P}[X = 1]$
Obtain (i) P[X=0] (ii) $\underline{P}[3 \leq X < 6]$ (iii) $\underline{P}[X > 0]$

Solution:

$$P[X=r] = \frac{\lambda^r e^{-\lambda}}{r!} \qquad r = 0,1,2\ldots$$

We find the parameter

$$P\ [X=2] = \frac{\lambda^2 e^{-1}}{2!} = \frac{0.8\,\lambda^1\ e^{-1}}{1!}$$

$$\lambda = 2 > 0.8$$
$$= 1.6$$

$$\int_0^1 f(2x)dx = 1$$

$$\frac{2x^2}{2}\ 1^2 - 0 = 1$$

The continuous Random Variable

1. The random variable X is continuous dif there exists a functio[n]
called the probability density function (pdf) such that

(i) $f(x) \geq 0$ for all x (tx)

(ii) $\int_{-\infty}^{\infty} f(x)dx = 1$

2. $\underline{P}\,[a < X < b] = \int_{-\infty}^{\infty} f(x)dx$ = area under the graph of f(x) from a

3. For the continuous r.v. X_2 $\underline{P}\,[X = x] = 0 \ \forall x$

4. The Distribution function F(x)

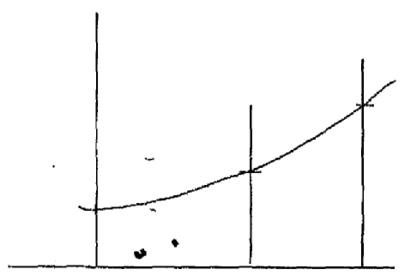
$$F(X) = \underline{P}\,[X \leq x] = \int_{-\infty}^{x} f(x)dx$$

Some properties of F(x)

(i) $0 \leq F(x) \leq 1$

(ii) F(x) is monotonic increasing

$x_1 < x_2 \Rightarrow F(x_1) < F(x_2)$.



(iii) $F(-\infty) = 0$

(iv) $F(+\infty) = 1$

Supposing

$X : \underline{P}\,[X = k] = < q^{k-1}$ $< = 40$

(i) Which of the following can be the pdf of cont. r.vs.

(i) $f(x) = 1 \quad 0 \le x \le 1$

(ii) $f(x) = 2x \quad 0 \le x \le 1$

(iii) $f(x) = x(x-2) \quad 0 \le x \le 1$

(iv) $f(x) = x(x3) \quad 0 \le x \le 5$

2. The r.v X has the pdf

$$f(x) = ax \quad 0 \le x \le 1$$
$$f(x) = a \quad 1 \le x \le 2$$
$$f(x) = ax + 3 \quad 2 \le x \le 3$$

elsewhere

Verify

$f(x) \ge 0$

$$\int_{-\infty}^{\infty} f(x)\, dx = 1$$

Obtain (i) a

(ii) $\underline{P}\,[X \le ½]$ (iii) $\underline{P}\,[X \ge 1.5]$

To calculate a

$$\int_{-\infty}^{\infty} f(x)dx = 1 \Rightarrow \quad f(x)\, dx = 1$$

$$\int_0^3 f(x)dx = \int_0^1 axdx + a\int_1^2 dx + \int_r^3 C\text{-}ax + 3a)dx = 1$$

$$= \left.\frac{ax^2}{2}\right]_0^1 + \left.ax\right]_1^2 + \left[\frac{-ax^2}{2} + 3ax\right]_2^3 = 1$$

$$= \frac{a}{2} + 2a - a + -ax^2 + \left(\frac{-9a}{2} + 9a + \frac{4a}{2} - 6a\right) = 1$$

$$\le \frac{a}{2} + 2a - a + -ax^2 + \frac{-9a}{2} + 9a + \frac{4a}{2} - 6a = 1$$

$$\frac{a + a - 2a - 9a + 18a + 4a - 12a}{a} = \frac{29a - 23a}{2}$$

$$= \frac{4a}{2} = 2a$$

$\Rightarrow$ $2a = 1$

$a = ½$

so $f(x) = \frac{x}{2}$ $0 \le x \le 1$

½ $0 \le x \le 1$

$-\frac{x}{2} + \frac{3}{2}$ $2 \le x \le 3$

θ elsewhere

$$P[X \le ½] = \int_0^{½} f(x)dx = \int_0^{½} \frac{x}{2}dx = \left.\frac{x^2}{4}\right]_0^1 = \frac{1}{16}$$

$$P[X \le 1.5] = \int_{1.5}^{3} f(x)dx = \int_{1.5}^{2} ½dx + \int_{1.5}^{3} \left[\frac{x}{C-2} + \frac{3}{d}\right] dx$$

$$= \left.\frac{x}{2}\right]_{1.5}^{2} + \left.\frac{-x^2}{4} + \frac{3x}{2}\right]_{2}^{3}$$

$$= 1 - \underline{3} + \left(-\frac{9}{4} + \frac{9}{2} + 1 - 3\right.$$

$$= 1 - {}^3/_4 - {}^9/_4 + {}^9/_2 + 1 - 3$$

$= ½$

$$\frac{2}{1} + \frac{9}{2} - \frac{3}{4} - \frac{9}{4} - \frac{3}{1}$$

$$\frac{8 + 18\ \ 3 - 9 - 12}{4}$$

Note that for all conts r.v.s

$P[a \le x \le b] = \underline{P}[a<X<b] = \underline{P}[a \le X<b] = \underline{P}[a \le X<b]$

$P[X \le 1] = \underline{P}[X<1]$

The Exponential Distribution E(λ)

The r.v. X is said to have the exponential dsn with parameter λ written E(λ), if its pdf is

$f(x) = \lambda e^{-\lambda x} \qquad x>0$

Example: The time between the arrival of successive cars at a road junction has the following pdf

$f(x) = \lambda e^{-0.5x} \qquad x>0$

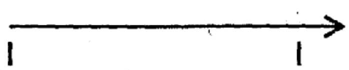
Obtain (i) the values of λ

(ii) The probability that no car comes within 1½ minutes of the least one.

Solution (i) To obtain λ.

$$\int_{-\infty}^{\infty} f(x)dx = 1 \qquad \Rightarrow \lambda = 0.5$$

(ii) $\underline{P}[X \le 1] =$



$$\int_0^1 f(x)dx = \int_0^1 0.5e^{-0.5x}\, dx$$

$$= -e^{-0.5x}\Big]_0^1 = 1-e^{-0.5x} = 1-e^{-0.5x} = 0.82$$

(iii) $\underline{P}[1 \le X \le 2.5]$

Function Of A Random Variable

Recall r.v. X is a fn from Ω to R. thus $X:\Omega \rightarrow R$

The function of the r.,v. X, g(x) is also a r.v.

$$\Omega \xrightarrow{X} \overset{x(w)}{R} \xrightarrow{g(x)} \overset{gx(w)}{R}$$

g(x) is thus a r.v. Our interest is to study the dsns and moments of such function.

Let Y = g(x) and F(y) The dsn of Y.

$F(Y) \;=\; \underline{P}[Y \leq y] \;=\; \underline{P}[g(x) \leq g^{-1}(y)]$

The above string of equation tells us that the dsn of g(x), F(y) can be found from that of x.

Example: $X:\cup(1,3)$

Find the pdf of (i) $Y = 3x+4$

(ii) $y = e^x$

Solution (1)

Let F(y) be the dsn of $Y = 3x+4$

$F(y) = \underline{P}\,[Y \leq y] = P[3x+4 \leq y]$

$$= \underline{P}\left[y \leq \frac{y-4}{3}\right]$$

$$= \frac{\frac{y-4}{3} - 1}{3-1} = \frac{y-7}{6}$$

Pdf of $y = f(y) = \frac{df(y)}{Dy} = \frac{1}{6} + \quad 7 \leq y \leq$

x max = 3

$$x = \frac{y-4}{3} = 3$$

$$\frac{y-4}{3} = 3$$

$y = 13$

$XMR = 1$

$x = \frac{y-4}{3} = 1$

$\frac{y-4}{} = 1$

$8y = 7$

Fns of a R.v.

1. X:E(0.3)
   Find the Pdf of Y = 2x-1
   X: b(n:p) find dsn of (i)Y = 2x
   (ii) Y = 3x

Solution:

$F(x) = (0.3)e^{-03x} \quad x \le 0$

$P[y \le y] = P[2x1 + \le y] = P[x \le \frac{y-1}{2}]$

$= \int_0^{\frac{y-1}{2}} f(x)\, dR = \int_0^{\frac{y-1}{2}} 0.3e^{-0.3x}\, dx = -e^{0.3x} \Big]_0^{\frac{y-1}{2}}$

$= 1 - e^{0.3(y½)}$

pdf of $Y = g(y) = \frac{0.3}{2} e^{0.3(y½)} = 0.15^{-065(y1)} \quad y \ge 1$

$x \le \frac{y-1}{2} \ge x = \frac{y-1}{2} \ge 0$

$y \ge 1$

, X b(n,P)

We find $Y = 2x$

$P[Y = r]$ $r = 0,2,4,6 \ldots .2n$

$P[Y = r] = [2x=r] = \underline{P} [X = {}^r/_2]$

${}^nC_{r/2}\ P^{r/2}\ q^{n-r/2}$

$/_2 = 0,1,2,3\ldots\ldots n$

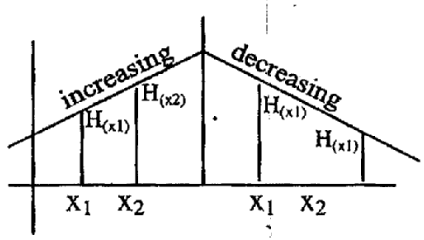
$r = 0,2,4 \ldots\ldots\ldots 2n$

Theorem: X is a conts r.v. with the pdf f(x).

$F(x) \geq 0$ $a \geq x \leq b$

Suppose $y = H(x)$ is a strictly monotone function of x and f(x) is differentiable. Then $Y = H(X)$ has the pdf g(y) given by $g(y) = f(x)\frac{dx}{dy}$ where x is expressed in terms of y.

$y = H(x)$ for monotone increasing $x_1 \leq x_2 \Rightarrow H(x_1) \leq H(x_2)$



Proof of Theorem

(1) Let Y be strictly increasing

i.e. $x_1 < x_2 \Rightarrow H(x_1) < H(x_2)$

Let G(y) be the dsn of Y. so that $g(y) = \frac{dG(y)}{dy}$

Then $G(y) = \underline{P}[Y \leq y] = \underline{P}LH(x) \leq y] = \underline{P}[x \leq X(y)] = F[H^{-1}(y)]$

Let $x = H^{-1}(y)$

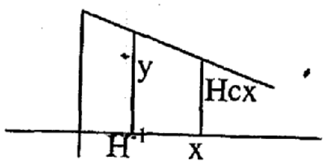
Then $G(y) = F(x)$

So $g(y) = dG(y) = \frac{dF(x)}{dx} \cdot \frac{dx}{dy} = f(x)\frac{dx}{dy}$

Let Y be strictly monotone decreasing

$x_1 < x_2 \leq H(x_1) > H(x_2)$

$G(y) = \underline{P}[Y \leq y] = \underline{P}[H(x) \leq y] = \underline{P}[x \geq H^{-1}(y)]$



Again if $x = H^{-1}(y)$

$$g(y) = \frac{dG(y)}{dy} = \frac{d[1-F(x)]}{} \cdot \frac{dx}{dy}$$

$$= -f(x)\frac{dx}{dy}$$

$\underline{P}(x) = P[x \leq x]$
$P[X \geq x]$
$= 1-\underline{P}[x \leq x]$

Combine the results of (i) and (ii) we have $g(y) = f(x)\left|\frac{dx}{dy}\right|$

Joint Dsn

1. Discrete Case

Let (xy) be a two dimensional r.v. X takes $X_1, X_2, \ldots$
Y takes Y_1 $Y_2 \ldots$.
(X,Y) take … (X_1,Y_1), $(X_2$ $Y_2)$

Let $P(x_i,yi)$ be the prob. That $(X,Y) = (x_i,yi)$, then $\{(x_i,yi)\ P(x_i,yi)$
i = 1,2 …} is called a two dimension dsn of the discrete ty

$P(x_i,yi) \geq 0$ i 2

x y

$\sum_j P(x_i\ y_i) = 1$ P(x,y)

(xy) . P(xy)

In dies

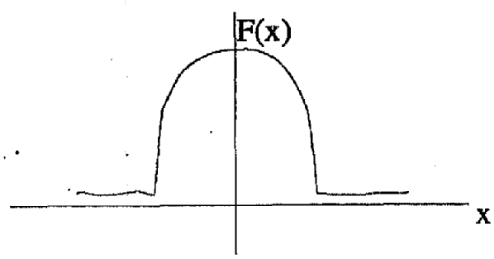
	1	2	3	4	5	6
1	11	12	13	14	15	16
2						
3						

2. The Continuous Case

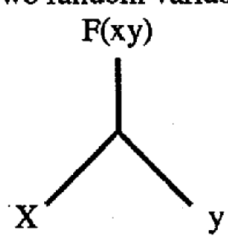
The two dimensional (xY) r.v. is a jointly distributed r.v. of th continuous type if there is a fuction of x and f(xy) set $f(x,y) \geq 0$

$$\iint F(x,y)\,dxdy = 1$$

The dimension of one random variable in two dimensional



The dimension of two random variable is three dimensional



Some properties of the joint dsn Define f(xy) = $\int_{-\infty}^{y}\int_{-\infty}^{x} f(x,y)\,dxdy$ =

F(xy) is the joint dsn function

(1) $F(-\infty, \infty) = 1$
(2) $F(-\infty, y) = 0$
(3) $F(-\infty, y) = 0 \;\forall\; y$
(4) $F(x, -\infty) = 0 \;\forall\; x$.

Example:
(i) X and Y are jointly distributed with the

$\iint$ joint pdf.

$F(x,y) = x^2 + Cxy \qquad o< x < 1$
(a) Find c
(b) P(Xs≤Y≤ ½   Y≤1)
(c) P [X> ½ , Y ≥ 1/2]

(a) To find c

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y)\, dxdy = 1$$

$$\rightarrow \int_0^2 \int_0^1 (x^2 + cxy)\, dxdy = 1$$

$$\int_0^2 \left({}^{x^3}/_3 + c^{x^2}/_2 y \right)\Bigg]_0^1 dy = \frac{y}{3} + \frac{cy^2}{4}\Bigg]_0^2$$

Integration with respect to x; integrate with respect to y.

$$\Rightarrow \tfrac{2}{3} + c = 1 \quad \Rightarrow c = \tfrac{1}{3}$$

so $f(x,y) = x^2 + \tfrac{1}{3}xy \quad \begin{matrix} 0 < x < 1 \\ 0 < y < 2 \end{matrix}$

$$P[X \le \tfrac{1}{2}, y \le 1] = \int_0^1 \int_0^{1/2} f(x,y)\, dxdy$$

$$= \int_0^1 \int_0^{1/2} (x^2 + \tfrac{1}{3}xy)\, dxdy$$

$$= \int_0^1 \left[\frac{x^3}{3} + \frac{x^2y}{6} \right]_0^{1/2} dy \text{ with r.t.x}$$

$$= \int_0^1 \left(\frac{1}{24} + \frac{y}{24}\right) dy = \left[\frac{y}{24} + \frac{y^2}{48} \right]_0^1 \text{ with r.t.y}$$

$$\frac{1}{24} + \frac{1}{48} = \frac{2+1}{48} = \frac{1}{16}$$

C_1 $P(x \ge \tfrac{1}{2}, y = 1/2) = \int_{1/2}^2 \int_{1/2}^1 f(x,y)dxdy$

$$= \int_{1/2}^2 \int_{1/2}^1 (x^2 + \tfrac{1}{3}xy)\, dxdy$$

$$= \int_{1/2}^{2} \left[\frac{x^3}{3} + \frac{x^3 y}{6} \right]_{1/2}^{1} dy \quad = \int_{1/2}^{2} \{(1/3 + y/6) - (1/24 + y/48)$$

integrate with r.t.y

$$= \left[\frac{y}{3} + \frac{y^2}{18} - \frac{y}{24} - \frac{y^2}{96} \right]_{1/2}^{2}$$

$$= (2/3 + 4/18 - 2/24 - 4/96) - 1/6 + 1/72 - 1/48 - 1/384) = 5/96$$

Examples: The jointly distributed r.v.s x and y have the ff joint

x \ y	0	1	2	3	4
1	c	c	2c	c	3c
2	2c	c	c	2c	4c
3	2c	3c	c	c	5c

Find (i) the value of c

(ii) $P(X = 1\ Y = 4)$

(iii) $P[X \leq 1, Y \leq 2]$

(iv) $\underline{P}[X \leq 2, Y \geq 3]$

(i) To find c

$$\Sigma P(XY) = 1 \Rightarrow c = 1/3$$

(ii) $3c = 3/30 = 1/10$

(iii) $P[X \leq 1, Y \leq 2] = P[x \leq 1,\ Y = 0,1,2]$

$= P[X \leq 1, Y = 0] + P[X \leq 1\ Y = 1] + P[X \leq 1, Y = 2]$

$= P[X = 1,\ Y = 0) + P[X = 1\ Y = 1] + P[X = 1,\ Y = 2]$

$$= \frac{1}{30} + \frac{1}{30} + \frac{2}{30} = \frac{4}{30} = \frac{2}{15}.$$

Assignment

1. $f(x,y) = 2 - x - y \qquad 0 < x < 1$

$\qquad\qquad\qquad\qquad\quad 0 < y < 1$

Obtain (i) $P[2x \le 1,\ y = ½]$

(ii) $P[x \ge 0.3,\ 0.2 \le Y \le 0.8]$

2. $f(x,y) = ae^{-0(x+y)} \qquad x>4,\ y>0$

(i) Find a

(ii) $P[X < ½,\ 1 \le y \le 2]$

Marginal Dsns

X and y are two jointly distributed r.v.s with pdf f(x,y). The marginal dsn of x, written h(x) and that of Y written g(y) are given by

$$h(x) = \int_{-\infty}^{\infty} f(xy)dy$$

$$g(y) = \int_{-\infty}^{\infty} f(x,y)dx$$

The graff of marginal dsn is two dimension if X and Y and (D) discrete with $\{(x_i,y_i)\ P(x_i,y_i)\}$ as the dsn. The marginal h(x) for x and g(yi) for Y are given by

$$h(x_i) = \sum_j P(x_i;y_i)$$

$$g(y_i) = \sum_g P(x_i;\ y_i)$$

We observed that the marginal pdf satisfy the following

$h(x) \ge 0 \qquad\qquad h(x_i) = 0$

$$\int_{-\infty}^{\infty} h(x)dx = 1 \quad \sum h(xi) = 1$$

Example: X and Y are jointly distributed with
$f(x,y) = c(6-x-y)$ $0<x<2$
$2<y<4$

Find

(i) The value of c
(ii) $P[1 \le X \le 2,\ Y \le 3]$
(iii) The margial $h(x)$, $g(y)$
(iv) $P[x \ge 1]$, $P[1 \le y \le 3]$, $P[y \ge 3]$

(i) $c = {}^1/_8$

(iii) $$h(x) = \int_{-\infty}^{\infty} f(x,y)dy$$

$$= {}^1/_8 \int_2^4 (6 - xy)\, dy$$

by-xy- $\frac{y^2}{2}$

$= {}^1/_8\ (6-2x) = ¼\ (3 - x)$ $0<x<2$

$$g(y) = \int_2^4 f(x,y)\, dx$$

$$= {}^1/_8 \int_2^4 (6-x-y)dx = {}^1/_8 \left[6x - x^2 - xy\right]_0^2$$

$= {}^1/_8\ [12 - 2 - 2y]$
$= ¼\ (5-y)$ $2<y<4$

(iv) P[x>1] $\int_1^2 h(x)dx \;=\; \frac{1}{4}\int_1^2 (3-x)dx$

$= \frac{1}{4}\left[3x - \frac{x^2}{2}\right]_1^2$

$= \frac{1}{4}\;(6-2-3+\frac{1}{2}) = \frac{3}{8}$

$P[\le y \le 3] \;=\; P\,[2 \le y \le 3]$

$= \int_2^3 <\;(x)\,dx \;=\; \frac{1}{4}\int_2^3 (5-y)$

(2) The jointly distributed r.v.s. X and Y have the pdf

$f(x,y) \;=\; k\;\; x - y \qquad 0<x<1$

$\qquad\qquad\qquad\qquad\qquad 0<y<1$

Find (i) k

(ii) h(x) g(y)

(iii) P(x c½, y≥½)

(iv) P(0.5<x≤2.3), P[0.5≤ Y≤0.8]

(3) The r.v.s X and Y have the dsn given below.

X \ Y	1	2	3	hx
0	.1	.3	.05	4.05
1	.05	c	2c	.05
2	.05	.09	.1	.2
g(y)	.01			

X \ Y	$y,\ y^2\ \ ym$	(hx)
x_1	P(x,y)	
x_2		
x_3		
x_n	$P(x_1y_1)$	$h(x_2)$
g(y)	$g(y)\ g(y_1)$	

Find (i) C

(ii) P[x≤1], P[Y≥3] P(2x≤2.2]

For the discrete case above we have.

(i) The row sum g(y) = (g(y_1) g(y_2)...g(ym)

= marginal dsn of y

(ii) The column of h(x) = h(x_1), h(x_2)...L(Xn)

= marginal dsn of x

Solution to (3)

(i) To fidnd C we recall that $\sum_{xy} P(x,y) = 1$

7 + 3cd = 1 C = 0.1

(ii) h(x) To obtain h(x), we sum each row and obtain h(x) {0,.4
1,0.35), (2, 0.2)}

g(y) to get g(y) we add each column and obtain

BIBLIOGRAPHY

Ryans, D.G. (1960); Characteristics of Teachers Nashinton: American council on Education.

Ndinechi G.I. (1990) A guide for Effective Typewriters Instruction in Secondary Schools Business Education Journal Volume II No. 2, Nigeria Association of Business Educators.

Adichie R. (1990): Introduction to College Statistics, Ibadan, Oxford University Press Ltd.

Akudolu, I. R. (1995); Effects of Computer Assisted Language Learning on Students Achievement and Interest in French. UNN (Unpublished Ph.D thesis).

Ali A. and Ohuche R. O. (1991): Teaching Senior Secondary School Mathematics Creatively. Onitsha: Summer Educational Pub. Ltd.

Harbor - Peters, V. F. (191); Target Task and Formal Methods of Presenting Secondary School Geometric Concepts. Their Effects on Retention. Josic Vol. 1 March.

James R. Flynn (1978): Humanism and Ideology London, Routledge and Kegan Paul Press.

Ohuche R.O. (1986); "Laboratory Approach to Science Teaching". Nigerian Journal of Education. Vol 3.

Onyejemezi, D.A. (1988):The Principles of Educational Technology. Onitsha Summer Educational Publishers.

Sinclair, L. R. (1990): Computer Science a Concise Instruction. Oxford Heninemann Newness Press.

Fredrick Klemm (1999) A History of Western Technology , London, JRR Press Ltd & Books.

Ozofor, N.M. (2000): Effects of Two Modes of Computer Aided Instruction on Students Achievements and Interest in Statistics and probability UNN (Unpublished Ph.D Thesis).

Ernest Nagel (1961): The Structure of Science London Oxford Press.

G. Vygosky (1990) Mathematical Handbook: Moscow, Mins Pub.

Wartofsky, M.W. (1968): Conceptual Foundations of Scientific Thou

Arthur Danto and Sidney Morgenbesser (1962): Philosophy of Scien Washington DC, Classic Papers Publishers.

John H. Randall, Modern Science (1980) London, Alistar C. Cro Press Ltd.

Huxley T H (1980) Popular Lectures in Science Subjects

Ernest Mach (1990): 20th century delsate in the Philosophy of Sci Boston, Die Mechnik.

Thomas S Kuhn (1970): The Structure of Scientific Revolution . N Imre Lakatos Press Ltd.

M. Bowden (1991): Science vs Evolution Great Britain, Bath Press, Avo

Spencer, F. Piltdown: A Scientific Forgery Oxford Press 1990.

Downie, N.M. and Heath, R. W. (1974) Basic Statistical Methods (4th Ed Harper and Row Pulishers. N/Y

Edwards, A.L. 9 (1972) Experimental Design in Psychological Researc (4th Ed.) Holt, Rinehart and Winston Inc. N/Y.

Guilford, J.P. (1965) Fundamental Statistics in Psychology and Education (4th Ed.) Mc Graw-Hill book company N/Y.

Hays, W.L. 9 (1973) Statistical for the Social Sciences (2nd Ed.) Halt, Rinehart and Winston, Inc. N / Y.

Marascuilo, L.A. (1971) Statistical Methods for Behavioural Science Research. MC Graw – Hill book company. N/Y.

Minimum, E.W. (1978) Statistical Reasoning in Psychology and Education (2nd Ed.) John Wiley and Sons N/Y.

Popham, W. and Sirotnik, K.A. (1973) Educational Statistics: Use and Interpretation (2nd Ed.) Harper and Row Publishers N/Y.

Robinson, P.W. (1981) Fundamentals Experimental Psychology. Prentice hall, Inc. Englewood cliffs.